Maths with Maple — Week 1 lab session

Introduction

1. Getting started

As soon as you enter the room, please log on to the network, and start up Maple. To do this, first double-click the "Load application menus" icon on the desktop. When this has finished (which may take a minute), click on the start menu, then "Applications", then "Academic", then "Mathematics", then "Classic Worksheet Maple 12".

In working through this sheet, and the lab sheets in subsequent weeks, you will need to consult the "Maple reference" notes. These were distributed in the first lecture, and also are available online at `http://www.shef.ac.uk/nps/MAS100`. If you see something in the lab sheet like **[8.6]**, that means that item 8.6 in the Maple notes is relevant.

2. Maple as a calculator

Exercise 2.1. Enter

```
2+2;
```

(As with almost all Maple commands, we need a semicolon at the end **[2.3]**. If Maple says "Warning, premature end of input", you probably left out the semicolon.) Then press the ENTER key. Maple will respond with

4

Now try the following calculation[1]:

```
(3 + 7 + 10) * (1000 - 8) /(900 + 90 + 2) - 17;
```

Note that we use `*` for multiplication[**5.1**]; it is a very common error to leave this out. If you ever see a message saying

Error, missing operator or ';'

then the most likely explanation is a missing `*`.

Exercise 2.2. Try the following calculations. In each case the answer will have some kind of pattern to it; see if you can explain it.

(a) $6^{20}\ 15^{20}/9^{20}$ (Enter this as `6^20*15^20/9^20;`)
(b) $(10^{10}-1)/99$
(c) $(10^{10}-10-9^2)/9^2$ (The explanation here is harder, and should be considered a challenge for enthusiasts only.)
(d) $(10^9+1)(10^{10}-10-9^2)/9^2$ (You should be able to explain how this relates to (c), even if you cannot explain (c) itself.) (The correct answer is *not* 1000000001/81; if you get that answer, you probably left out a `*`.)

Maple will usually leave answers as fractions (or other exact mathematical expressions) rather than giving an approximate numerical answer. If you want a numerical answer, you can use the function `evalf()`. For example:

```
1 + 1/2 + 1/3 + 1/4;
```

25/12

```
evalf(1 + 1/2 + 1/3 + 1/4);
```

2.083333333

(A slightly more efficient approach here is to enter `1+1/2+1/3+1/4;`, and then `evalf(%);`. The `%` symbol refers to the last thing that Maple worked out **[2.5]**.)

Exercise 2.3. Calculate the following as exact fractions, then as numerical approximations. What do you see?

(a) $3+\dfrac{1}{7}$ (b) $3+\dfrac{1}{7+\dfrac{1}{15}}$ (c) $3+\dfrac{1}{7+\dfrac{1}{15+\dfrac{1}{1+\dfrac{1}{293}}}}$

You need to be very careful with brackets when entering these expressions. For example, the last one should be

```
3 + 1/(7 + 1/(15 + 1/(1 + 1/293)));
```

[1] From the Beaver's Lesson: `http://tinyurl.com/2dv793`, verses 16–17

Exercise 2.4. Maple's notation for the constant $e \simeq 2.718281828$ (the base of natural logarithms) is `exp(1)` (see [**7.3**]). Read [**3.2,3.3**] to find out how to calculate 4997 digits of e. (You should notice that the last 5 digits are all the same; this is the first time that this happens.) [2]

Exercise 2.5. It is a remarkable fact that $e^{\pi\sqrt{163}}$ is extremely close to being an integer (for a reason involving some very advanced mathematics). Let us check this.

(a) Tell Maple to do all calculations to 40 digits (see [**3.3**]).
(b) Enter `x:=exp(Pi*sqrt(163));`. Maple will display this in traditional notation, but will not evaluate it numerically. For the syntax used here, see [**2.7,7.3,7.1,5.9**].
(c) Enter `y:=evalf(x);`. Maple will print this in scientific notation, which is not too helpful for our present purposes. To display it in a more useful way, enter

```
printf("%20.20f\n\n",y);
```

(Do not worry about the details of this, as we will not use the `printf` command again.) You should see that there is a long string of 9's after the decimal point, showing that y is just a tiny bit less than an integer.
(d) To find the integer nearest to y, enter `z:=round(y);`. (Of course this can just be read off from (c), but we prefer not to have to retype it.) Now calculate $y - z$, to see that y is less than 10^{-12} away from an integer.
(e) Enter `restart;`. This removes the definitions of x, y and z, which would otherwise cause trouble in later exercises.

3. Symbolic algebra

Exercise 3.1. Enter

```
A := (x^2-4*y^2)*(x^3-x*y^2);
```

(If Maple prints this as a horrible expression involving $e^{\pi\sqrt{163}}$, then you probably forgot to remove the definitions of x, y and z made in the previous exercise. Enter `restart;` and try again.)

Now try `simplify(A);`, `expand(A);`, `factor(A);` and `convert(expand(A),horner,x);`. Which of these forms do you think is the most illuminating? (There is no right answer.)

Exercise 3.2. Ask Maple to simplify the expression

$$\frac{2x}{x^2-1} + \frac{1}{x+x^2} + \frac{1}{x-x^2}.$$

Now see if you can get the same answer by hand.

4. Plotting

Exercise 4.1. Ask Maple to plot the graph of $y = \sin(x)$ from $x = -4$ to $x = 4$ (see [**11.1**] for the syntax). Now plot the function

$$y = x - x^3/6 + x^5/120 - x^7/5040$$

from $x = -4$ to $x = 4$. What do you notice? Plot the two curves together, as explained in [**11.5**].

5. Solving

Exercise 5.1. Ask Maple to solve the equations

$$x^2 + y^2 = 1$$
$$(x-1)^2 + (y-1)^2 = 1,$$

like this:

```
solve({x^2+y^2=1,(x-1)^2+(y-1)^2=1},{x,y});
```

Can you find the solution by hand? Can you draw (by hand) the curves $x^2 + y^2 = 1$ and $(x-1)^2 + (y-1)^2 = 1$, and thus obtain the solution graphically?

6. Calculus

Exercise 6.1. Read [**12.1,12.2**]. Then ask Maple to differentiate the function $\ln(\ln(\ln(x)))$ with respect to x. Now differentiate the functions $(3x+4)/(2x+3)$ and $(1 + x^2 + x^4/2)e^{-x^2}$, and simplify your answers.

Exercise 6.2. Ask Maple to evaluate the integral $\int_0^1 \sqrt{1-x^2}\,dx$, as explained in [**13.3**]

[2] You may think that calculating thousands of digits of e is a useless activity, but it could get you a job at Google: see `http://tinyurl.com/ygat93`