Maths with Maple — Week 8 lab session

Integration 2

This sheet deliberately gives very little help with Maple syntax, and the same will be true for the remaining sheets. As you will see from the past papers on the course website, you will need a fairly detailed knowledge of the syntax for the exam.

Exercise 1. Enter the following definitions:

$$p_0 = \sqrt{\tfrac{1}{2}} \qquad p_1 = \sqrt{\tfrac{3}{2}}x \qquad p_2 = \sqrt{\tfrac{5}{8}}(3x^2-1) \qquad p_3 = \sqrt{\tfrac{7}{8}}(5x^3-3x)$$

(Remember that p_2 should be entered as `p[2]`, and so on.) Find $\int_{-1}^{1} p_i p_j \, dx$ for various i and j (including the case $i = j$). What pattern do you observe? There is a function p_4 of the form $ax^4 + bx^2 + c$ (with $a > 0$) that makes the pattern continue. What are a, b and c? (The first step is to define $p_4 = ax^4 + bx^2 + c$, leaving a, b and c as variables. Then calculate $\int_{-1}^{1} p_2 p_4 \, dx$ and so on, giving answers that depend on a, b and c. This should give you some equations relating a, b and c, which you should solve. If you use the `solve` command, you may find it helps to set `_EnvExplicit:=true` first.)

Now enter the definition

$$q(n) = \frac{\sqrt{n+1/2}}{2^n\,(n!)} \frac{d^n}{dx^n}\left((x^2-1)^n\right).$$

Check that $q(n) = p_n$ for $n = 1, \ldots, 4$.

Exercise 2. For various positive integers n and m, calculate and factorise the integral $\int x^n \ln(x)^m \, dx$. What terms appear in the answer? How does this depend on n and m? Write down your conclusions as a self-contained statement that would make sense to someone who had not read the question.

Exercise 3. You should be aware that there are many integrals for which the answer simply cannot be written in terms of familiar functions. In same cases, Maple will write the answer in terms of more obscure functions instead. For example, try the following:

$$\textbf{(a)} \int e^{-x^2} \, dx \qquad \textbf{(b)} \int \frac{dx}{\ln(x)} \qquad \textbf{(c)} \int \frac{dx}{\sqrt{1-x^2}\sqrt{1-2x^2}} \qquad \textbf{(d)} \int (x^8+1)^{-1/2} \, dx$$

In some other cases, Maple will just give up. For example, try the following:

$$\textbf{(e)} \int \sin(x)\ln(\ln(x)) \, dx \qquad \textbf{(f)} \int \sin(\sin(\sin(x))) \, dx \qquad \textbf{(g)} \int \frac{dx}{\sqrt{1+x+x^{10}}}$$

What is the simplest function that you can find that makes Maple give up? (There is one that you can enter with just three characters.)

Exercise 4. Consider the function

$$y = \sin(x) + \tfrac{1}{3}\sin(3x) + \tfrac{1}{5}\sin(5x) + \tfrac{1}{7}\sin(7x).$$

Plot y and $\int y \, dx$. Describe and explain the relationship between the shapes of these two graphs.

Exercise 5. Put $y = x\,e^{-x}\sin(20x)$ and $z = 20\int y \, dx$. Plot y and z for various ranges of x. Describe in detail, and explain, the relationship between the two graphs. Then plot $y^2 + z^2$. This is very close to a function with a much simpler formula; work out what it is.

Exercise 6. Find a, b and c such that $\int_0^\infty x^k(ax^2+bx+c)e^{-x} \, dx = k$ for $k = 1, 2$ and 3.

Maple syntax revision

Exercise 7. Find an approximate solution to $x(\ln(\ln(x)) + \ln(x) - 1) = 541$ close to $x = 100$.

Exercise 8. What would you enter to generate the sequence $a + b^2, a + b^3, a + b^4, \ldots, a + b^{10}$?

Exercise 9. Plot the curve $x^4 + y^4 = 1$ together with the curve given parametrically by $x = \cos(t)/(2 + \cos(4t))$ and $y = \sin(t)/(2 + \cos(4t))$.

Exercise 10. Find π^{76}/e^{87} to 100 decimal places.

Exercise 11. Simplify the expression $\sin(2x)\tan(2x)\dfrac{d^2}{dx^2}\log(\tan(x))$.