Maths with Maple — Week 9 lab session

Revision

Please refer to the "Notes on Maple syntax" as you do these questions. If you do not have a paper copy with you, then visit `http://www.shef.ac.uk/nps/MAS100/notes/primer.pdf`. Make sure you have a paper copy to consult when revising over the Christmas break.

Exercise 1. Enter the definitions $x_1 = \sin(\theta)\sin(\phi)$, $x_2 = \sin(\theta)\cos(\phi)$, and $x_3 = \cos(\theta)$. Simplify $x_1^2 + x_2^2 + x_3^2$.

Exercise 2. Enter the definition

$$u = 8 - (x-y)^2(x+y)^2(16 - 4x^2 - y^4).$$

Substitute $x = 2\cos(t)$ and $y = 2\sin(t)$ in u and simplify the result. Try both the `simplify()` command and the `combine()` command.

Exercise 3. Solve the equations

$$\begin{aligned} ax + by + cz &= 1 \\ ay + bz + cx &= 1 \\ az + bx + cy &= 1 \end{aligned}$$

to find x, y and z in terms of a, b and c.

Exercise 4. Find an approximate solution of $x^4 + \sin(x) = 10^4$ close to $x = 10$. Arrange your syntax so that Maple gives you an equation, not just a number.

Exercise 5. Enter the definition $f(x) = (e^x + x^7)/(e^x - x^7)$. Find a numerical approximation to $f(5)$. Then find numerical approximations for the whole sequence $f(0), f(1), \ldots, f(50)$.

Exercise 6. Plot the curve $x^4 + y^4 = 4$, together with the curve given by $x = (2 + \sin(8t))\cos(t)$ and $y = (2 + \sin(8t))\sin(t)$ (for $0 \le t \le 2\pi$).

Exercise 7. Plot $y = \tan(\pi x)$, $y = \cot(\pi x)$, $y = -1$ and $y = 1$ on the same graph, for x from -4 to 4. Tell Maple to restrict the vertical range from -2 to 2, to use the same scale on the two axes, and to skip over discontinuities.

Exercise 8. Enter the definitions

$$\begin{aligned} f(t) &= \frac{(2+\sqrt{3})t - 1}{2 + \sqrt{3} + t} \\ g(t) &= f(f(f(t))) \\ h(t) &= g(g(t)). \end{aligned}$$

Simplify $g(t)$ and $h(t)$.

Exercise 9. Find and simplify $\left(\frac{1}{y}\frac{d^8y}{dx^8}\right)^{1/8}$, where $y = \sin(10x)$.

Exercise 10. Plot the curves $y = n^{3/2}x^n(1-x)^2$ for $n = 1, \ldots, 10$ on the same graph, with x running from 0 to 1. Ideally you should use the `seq()` command, otherwise you will have to do a lot of typing.

Exercise 11. Put $y = \dfrac{x^4(1-x)^4}{1+x^2}$. Find the indefinite integral $\int y\,dx$, the definite integral $\int_0^1 y\,dx$ and the derivative dy/dx. Plot the graph of y from $x = 0$ to $x = 1$; you should see a single hump. Find numerical approximations to the position and height of the hump.

Exercise 12. Define $a(n)$ to be the approximate numerical value of $(1+1/n)^n/e$. Find the sequence $a(1), a(2), \ldots, a(100)$. What is the first n for which $a(n) > 0.99$?