Integration 3

Integration by substitution — I

- Consider $\int \frac{\sin(x)}{\cos(x)^n}\,dx$.

- Consider $\int \frac{\sin(x)}{\cos(x)^n}\, dx$.
- Put $u = \cos(x)$

- To find $\int f(x)\, dx$, pick out some part of $f(x)$ and call it u.

Integration by substitution — I

- Consider $\displaystyle\int \frac{\sin(x)}{\cos(x)^n}\,dx$.
- Put $u = \cos(x)$, so $du/dx = -\sin(x)$

- To find $\int f(x)\,dx$, pick out some part of $f(x)$ and call it u.
- Find du/dx

Integration by substitution — I

- Consider $\displaystyle\int \frac{\sin(x)}{\cos(x)^n}\,dx$.
- Put $u = \cos(x)$, so $du/dx = -\sin(x)$, so $dx = -du/\sin(x)$

- To find $\int f(x)\,dx$, pick out some part of $f(x)$ and call it u.
- Find du/dx, and rearrange to express dx in terms of x and du.

Integration by substitution — I

- Consider $\displaystyle\int \frac{\sin(x)}{\cos(x)^n}\,dx$.
- Put $u = \cos(x)$, so $du/dx = -\sin(x)$, so $dx = -du/\sin(x)$
- $$\int \frac{\sin(x)}{\cos(x)^n}\,dx = \int \frac{\sin(x)}{u^n}\,\frac{-du}{\sin(x)}$$

- To find $\int f(x)\,dx$, pick out some part of $f(x)$ and call it u.
- Find du/dx, and rearrange to express dx in terms of x and du.
- Rewrite the integral in terms of u and du.

Integration by substitution — I

- Consider $\int \frac{\sin(x)}{\cos(x)^n}\, dx$.
- Put $u = \cos(x)$, so $du/dx = -\sin(x)$, so $dx = -du/\sin(x)$
- $$\int \frac{\sin(x)}{\cos(x)^n}\, dx = \int \frac{\sin(x)}{u^n} \frac{-du}{\sin(x)} = -\int u^{-n}\, du$$

- To find $\int f(x)\, dx$, pick out some part of $f(x)$ and call it u.
- Find du/dx, and rearrange to express dx in terms of x and du.
- Rewrite the integral in terms of u and du.

Integration by substitution — I

- Consider $\int \frac{\sin(x)}{\cos(x)^n}\,dx$.
- Put $u = \cos(x)$, so $du/dx = -\sin(x)$, so $dx = -du/\sin(x)$
- $$\int \frac{\sin(x)}{\cos(x)^n}\,dx = \int \frac{\sin(x)}{u^n}\frac{-du}{\sin(x)} = -\int u^{-n}\,du$$
$$= u^{1-n}/(n-1)$$

- To find $\int f(x)\,dx$, pick out some part of $f(x)$ and call it u.
- Find du/dx, and rearrange to express dx in terms of x and du.
- Rewrite the integral in terms of u and du.
- Evaluate the integral

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- $$\int \frac{\sin(x)}{\cos(x)^n}\,dx = \int \frac{\sin(x)}{u^n}\,\frac{-du}{\sin(x)} = -\int u^{-n}\,du$$
  $$= u^{1-n}/(n-1) = \frac{\cos(x)^{1-n}}{n-1}$$

- To find $\int f(x)\,dx$, pick out some part of $f(x)$ and call it u.
- Find du/dx, and rearrange to express dx in terms of x and du.
- Rewrite the integral in terms of u and du.
- Evaluate the integral, then rewrite the result in terms of x.

Integration by substitution — II

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- Consider $\int xe^{-4x^2}\,dx$.

Integration by substitution — II

- Consider $\int xe^{-4x^2}\,dx$.
- Put $u = -4x^2$

- To find $\int f(x)\,dx$, pick out some part of $f(x)$ and call it u.

Integration by substitution — II

- Consider $\int xe^{-4x^2}\,dx$.
- Put $u = -4x^2$, so $du/dx = -8x$

- To find $\int f(x)\,dx$, pick out some part of $f(x)$ and call it u.
- Find du/dx

Integration by substitution — II

- Consider $\int xe^{-4x^2}\,dx$.
- Put $u = -4x^2$, so $du/dx = -8x$, so $dx = -du/(8x)$

- To find $\int f(x)\,dx$, pick out some part of $f(x)$ and call it u.
- Find du/dx, and rearrange to express dx in terms of x and du.

Integration by substitution — II

- Consider $\int xe^{-4x^2}\,dx$.
- Put $u = -4x^2$, so $du/dx = -8x$, so $dx = -du/(8x)$
- $$\int xe^{-4x^2}\,dx = \int -xe^{u}\frac{du}{8x}$$

- To find $\int f(x)\,dx$, pick out some part of $f(x)$ and call it u.
- Find du/dx, and rearrange to express dx in terms of x and du.
- Rewrite the integral in terms of u and du.

- Consider $\int xe^{-4x^2}\,dx$.
- Put $u = -4x^2$, so $du/dx = -8x$, so $dx = -du/(8x)$
- $$\int xe^{-4x^2}\,dx = \int -xe^u \frac{du}{8x} = -\frac{1}{8}\int e^u\,du$$

- To find $\int f(x)\,dx$, pick out some part of $f(x)$ and call it u.
- Find du/dx, and rearrange to express dx in terms of x and du.
- Rewrite the integral in terms of u and du.

Integration by substitution — II

- Consider $\int xe^{-4x^2}\,dx$.
- Put $u = -4x^2$, so $du/dx = -8x$, so $dx = -du/(8x)$
- $$\int xe^{-4x^2}\,dx = \int -xe^{u}\frac{du}{8x} = -\frac{1}{8}\int e^{u}\,du$$
  $$= -e^{u}/8$$

- To find $\int f(x)\,dx$, pick out some part of $f(x)$ and call it u.
- Find du/dx, and rearrange to express dx in terms of x and du.
- Rewrite the integral in terms of u and du.
- Evaluate the integral

Integration by substitution — II

- Consider $\int xe^{-4x^2}\,dx$.
- Put $u = -4x^2$, so $du/dx = -8x$, so $dx = -du/(8x)$
- $$\int xe^{-4x^2}\,dx = \int -xe^u \frac{du}{8x} = -\frac{1}{8}\int e^u\,du$$
$$= -e^u/8 = -e^{-4x^2}/8$$

- To find $\int f(x)\,dx$, pick out some part of $f(x)$ and call it u.
- Find du/dx, and rearrange to express dx in terms of x and du.
- Rewrite the integral in terms of u and du.
- Evaluate the integral, then rewrite the result in terms of x.

Integration by substitution — III

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- Consider $\displaystyle\int \frac{dx}{4x^2 + 4x + 2}$.

- Consider $\int \frac{dx}{4x^2+4x+2} = \int \frac{dx}{(2x+1)^2+1}$.
- Put $u = 2x+1$

- To find $\int f(x)\,dx$, pick out some part of $f(x)$ and call it u.

- Consider $\int \frac{dx}{4x^2 + 4x + 2} = \int \frac{dx}{(2x + 1)^2 + 1}$.
- Put $u = 2x + 1$, so $du/dx = 2$

- To find $\int f(x)\,dx$, pick out some part of $f(x)$ and call it u.
- Find du/dx

Integration by substitution — III

- Consider $\displaystyle\int \frac{dx}{4x^2+4x+2} = \int \frac{dx}{(2x+1)^2+1}$.
- Put $u = 2x+1$, so $du/dx = 2$, so $dx = du/2$

- To find $\int f(x)\,dx$, pick out some part of $f(x)$ and call it u.
- Find du/dx, and rearrange to express dx in terms of x and du.

Integration by substitution — III

- Consider $\int \frac{dx}{4x^2+4x+2} = \int \frac{dx}{(2x+1)^2+1}$.
- Put $u = 2x+1$, so $du/dx = 2$, so $dx = du/2$
- $$\int \frac{dx}{4x^2+4x+2} = \int \frac{du/2}{u^2+1}$$

- To find $\int f(x)\,dx$, pick out some part of $f(x)$ and call it u.
- Find du/dx, and rearrange to express dx in terms of x and du.
- Rewrite the integral in terms of u and du.

Integration by substitution — III

- Consider $\int \frac{dx}{4x^2+4x+2} = \int \frac{dx}{(2x+1)^2+1}$.
- Put $u = 2x+1$, so $du/dx = 2$, so $dx = du/2$
- $$\begin{aligned}\int \frac{dx}{4x^2+4x+2} &= \int \frac{du/2}{u^2+1} \\ &= \arctan(u)/2\end{aligned}$$

- To find $\int f(x)\,dx$, pick out some part of $f(x)$ and call it u.
- Find du/dx, and rearrange to express dx in terms of x and du.
- Rewrite the integral in terms of u and du.
- Evaluate the integral

Integration by substitution — III

- Consider $\int \frac{dx}{4x^2+4x+2} = \int \frac{dx}{(2x+1)^2+1}$.
- Put $u = 2x+1$, so $du/dx = 2$, so $dx = du/2$
- $$\int \frac{dx}{4x^2+4x+2} = \int \frac{du/2}{u^2+1}$$
  $$= \arctan(u)/2 = \arctan(2x+1)/2$$

- To find $\int f(x)\,dx$, pick out some part of $f(x)$ and call it u.
- Find du/dx, and rearrange to express dx in terms of x and du.
- Rewrite the integral in terms of u and du.
- Evaluate the integral, then rewrite the result in terms of x.

Integration by substitution — IV

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- Consider $\displaystyle\int \frac{dx}{\sqrt{x - x^2}}$.

- Consider $\int \frac{dx}{\sqrt{x - x^2}}$.
- Put $x = t^2$

- To find $\int f(x)\, dx$, put x equal to some function of t.

Integration by substitution — IV

- Consider $\int \frac{dx}{\sqrt{x - x^2}}$.
- Put $x = t^2$, so $dx/dt = 2t$

- To find $\int f(x)\, dx$, put x equal to some function of t.
- Find dx/dt

Integration by substitution — IV

- Consider $\int \frac{dx}{\sqrt{x - x^2}}$.
- Put $x = t^2$, so $dx/dt = 2t$, so $dx = 2t\,dt$

- To find $\int f(x)\,dx$, put x equal to some function of t.
- Find dx/dt, and rearrange to express dx in terms of t and dt.

Integration by substitution — IV

- Consider $\int \frac{dx}{\sqrt{x - x^2}}$.
- Put $x = t^2$, so $dx/dt = 2t$, so $dx = 2t\,dt$

$$\sqrt{x - x^2} = \sqrt{t^2 - t^4} = t\sqrt{1 - t^2}$$

- To find $\int f(x)\,dx$, put x equal to some function of t.
- Find dx/dt, and rearrange to express dx in terms of t and dt.
- Rewrite the integral in terms of t and dt.

Integration by substitution — IV

- Consider $\int \frac{dx}{\sqrt{x-x^2}}$.
- Put $x = t^2$, so $dx/dt = 2t$, so $dx = 2t\,dt$

$$\sqrt{x-x^2} = \sqrt{t^2-t^4} = t\sqrt{1-t^2}$$
$$\int \frac{dx}{\sqrt{x-x^2}} = \int \frac{2t\,dt}{t\sqrt{1-t^2}}$$

- To find $\int f(x)\,dx$, put x equal to some function of t.
- Find dx/dt, and rearrange to express dx in terms of t and dt.
- Rewrite the integral in terms of t and dt.

Integration by substitution — IV

- Consider $\int \frac{dx}{\sqrt{x - x^2}}$.
- Put $x = t^2$, so $dx/dt = 2t$, so $dx = 2t\,dt$

$$\sqrt{x - x^2} = \sqrt{t^2 - t^4} = t\sqrt{1 - t^2}$$
$$\int \frac{dx}{\sqrt{x - x^2}} = \int \frac{2t\,dt}{t\sqrt{1 - t^2}} = 2\int \frac{dt}{\sqrt{1 - t^2}}$$

- To find $\int f(x)\,dx$, put x equal to some function of t.
- Find dx/dt, and rearrange to express dx in terms of t and dt.
- Rewrite the integral in terms of t and dt.

Integration by substitution — IV

- Consider $\displaystyle\int \frac{dx}{\sqrt{x - x^2}}$.
- Put $x = t^2$, so $dx/dt = 2t$, so $dx = 2t\,dt$

$$\sqrt{x - x^2} = \sqrt{t^2 - t^4} = t\sqrt{1 - t^2}$$

$$\int \frac{dx}{\sqrt{x - x^2}} = \int \frac{2t\,dt}{t\sqrt{1 - t^2}} = 2\int \frac{dt}{\sqrt{1 - t^2}}$$

$$= 2\arcsin(t)$$

- To find $\int f(x)\,dx$, put x equal to some function of t.
- Find dx/dt, and rearrange to express dx in terms of t and dt.
- Rewrite the integral in terms of t and dt.
- Evaluate the integral

Integration by substitution — IV

- Consider $\displaystyle\int \frac{dx}{\sqrt{x - x^2}}$.
- Put $x = t^2$, so $dx/dt = 2t$, so $dx = 2t\,dt$

$$\sqrt{x - x^2} = \sqrt{t^2 - t^4} = t\sqrt{1 - t^2}$$

$$\int \frac{dx}{\sqrt{x - x^2}} = \int \frac{2t\,dt}{t\sqrt{1 - t^2}} = 2\int \frac{dt}{\sqrt{1 - t^2}}$$

$$= 2\arcsin(t) = 2\arcsin(\sqrt{x})$$

- To find $\int f(x)\,dx$, put x equal to some function of t.
- Find dx/dt, and rearrange to express dx in terms of t and dt.
- Rewrite the integral in terms of t and dt.
- Evaluate the integral, then rewrite the result in terms of x.

Integration by substitution — V

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- Consider $\int \log(x)^2 \, dx$.

Integration by substitution — V

- Consider $\displaystyle\int \log(x)^2\, dx$.
- Put $x = e^t$

- To find $\int f(x)\, dx$, put x equal to some function of t.

Integration by substitution — V

- Consider $\displaystyle\int \log(x)^2\,dx$.
- Put $x = e^t$, so $dx/dt = e^t$

- To find $\int f(x)\,dx$, put x equal to some function of t.
- Find dx/dt

Integration by substitution — V

- Consider $\displaystyle\int \log(x)^2\,dx$.
- Put $x = e^t$, so $dx/dt = e^t$, so $dx = e^t\,dt$

- To find $\int f(x)\,dx$, put x equal to some function of t.
- Find dx/dt, and rearrange to express dx in terms of t and dt.

Integration by substitution — V

- Consider $\int \log(x)^2 \, dx$.
- Put $x = e^t$, so $dx/dt = e^t$, so $dx = e^t \, dt$

$$\int \log(x)^2 \, dx = \int \log(e^t)^2 e^t \, dt$$

- To find $\int f(x) \, dx$, put x equal to some function of t.
- Find dx/dt, and rearrange to express dx in terms of t and dt.
- Rewrite the integral in terms of t and dt.

Integration by substitution — V

- Consider $\displaystyle\int \log(x)^2\, dx$.
- Put $x = e^t$, so $dx/dt = e^t$, so $dx = e^t\, dt$

$$\int \log(x)^2\, dx = \int \log(e^t)^2 e^t\, dt = \int t^2 e^t\, dt$$

- To find $\int f(x)\, dx$, put x equal to some function of t.
- Find dx/dt, and rearrange to express dx in terms of t and dt.
- Rewrite the integral in terms of t and dt.

Integration by substitution — V

- Consider $\int \log(x)^2 \, dx$.
- Put $x = e^t$, so $dx/dt = e^t$, so $dx = e^t \, dt$

$$\int \log(x)^2 \, dx = \int \log(e^t)^2 e^t \, dt = \int t^2 e^t \, dt$$
$$= (t^2 - 2t + 2)e^t$$

- To find $\int f(x) \, dx$, put x equal to some function of t.
- Find dx/dt, and rearrange to express dx in terms of t and dt.
- Rewrite the integral in terms of t and dt.
- Evaluate the integral

Integration by substitution — V

- Consider $\int \log(x)^2\,dx$.
- Put $x = e^t$, so $dx/dt = e^t$, so $dx = e^t\,dt$

$$\int \log(x)^2\,dx = \int \log(e^t)^2 e^t\,dt = \int t^2 e^t\,dt$$
$$= (t^2 - 2t + 2)e^t = (\log(x)^2 - 2\log(x) + 2)x$$

- To find $\int f(x)\,dx$, put x equal to some function of t.
- Find dx/dt, and rearrange to express dx in terms of t and dt.
- Rewrite the integral in terms of t and dt.
- Evaluate the integral, then rewrite the result in terms of x.

Examples I

Examples I

- $\int \tan(x)\, dx$

Examples I

- $\int \tan(x)\, dx = \int \frac{\sin(x)}{\cos(x)}\, dx$

Examples I

- $\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx = -\int \frac{\cos'(x)}{\cos(x)}\,dx$

Examples I

- $\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx = -\int \frac{\cos'(x)}{\cos(x)}\,dx = -\log(\cos(x)).$

Examples I

- $$\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx = -\int \frac{\cos'(x)}{\cos(x)}\,dx = -\log(\cos(x)).$$
- Consider $\int x^2 \tan(x^3)\,dx$.

Examples I

- $\displaystyle\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx = -\int \frac{\cos'(x)}{\cos(x)}\,dx = -\log(\cos(x)).$
- Consider $\int x^2 \tan(x^3)\,dx$. Put $u = x^3$, so $du = 3x^2\,dx$, so $dx = du/(3x^2)$.

Examples I

- $\displaystyle\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx = -\int \frac{\cos'(x)}{\cos(x)}\,dx = -\log(\cos(x)).$
- Consider $\int x^2 \tan(x^3)\,dx$. Put $u = x^3$, so $du = 3x^2\,dx$, so $dx = du/(3x^2)$.

$$\int x^2 \tan(x^3)\,dx = \int x^2 \tan(u) \frac{du}{3x^2}$$

Examples I

- $\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx = -\int \frac{\cos'(x)}{\cos(x)}\,dx = -\log(\cos(x))$.
- Consider $\int x^2 \tan(x^3)\,dx$. Put $u = x^3$, so $du = 3x^2\,dx$, so $dx = du/(3x^2)$.

$$\int x^2 \tan(x^3)\,dx = \int x^2 \tan(u)\frac{du}{3x^2} = \frac{1}{3}\int \tan(u)\,du$$

Examples I

- $\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx = -\int \frac{\cos'(x)}{\cos(x)}\,dx = -\log(\cos(x))$.
- Consider $\int x^2 \tan(x^3)\,dx$. Put $u = x^3$, so $du = 3x^2\,dx$, so $dx = du/(3x^2)$.

$$\int x^2 \tan(x^3)\,dx = \int x^2 \tan(u)\frac{du}{3x^2} = \frac{1}{3}\int \tan(u)\,du = -\log(\cos(u))/3$$

Examples I

- $\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx = -\int \frac{\cos'(x)}{\cos(x)}\,dx = -\log(\cos(x))$.
- Consider $\int x^2 \tan(x^3)\,dx$. Put $u = x^3$, so $du = 3x^2\,dx$, so $dx = du/(3x^2)$.

$$\int x^2 \tan(x^3)\,dx = \int x^2 \tan(u) \frac{du}{3x^2} = \frac{1}{3}\int \tan(u)\,du = -\log(\cos(u))/3$$
$$= -\log(\cos(x^3))/3$$

Examples I

- $\displaystyle\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx = -\int \frac{\cos'(x)}{\cos(x)}\,dx = -\log(\cos(x)).$
- Consider $\int x^2 \tan(x^3)\,dx$. Put $u = x^3$, so $du = 3x^2\,dx$, so $dx = du/(3x^2)$.

$$\begin{aligned}\int x^2 \tan(x^3)\,dx &= \int x^2 \tan(u)\frac{du}{3x^2} = \frac{1}{3}\int \tan(u)\,du = -\log(\cos(u))/3 \\ &= -\log(\cos(x^3))/3\end{aligned}$$

- Consider $\int x e^{\sqrt{x}}\,dx$.

Examples I

- $\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx = -\int \frac{\cos'(x)}{\cos(x)}\,dx = -\log(\cos(x))$.
- Consider $\int x^2 \tan(x^3)\,dx$. Put $u = x^3$, so $du = 3x^2\,dx$, so $dx = du/(3x^2)$.

$$\begin{aligned}\int x^2 \tan(x^3)\,dx &= \int x^2 \tan(u)\frac{du}{3x^2} = \frac{1}{3}\int \tan(u)\,du = -\log(\cos(u))/3 \\ &= -\log(\cos(x^3))/3\end{aligned}$$

- Consider $\int xe^{\sqrt{x}}\,dx$. Put $t = \sqrt{x}$, so $x = t^2$, so $dx = 2t\,dt$.

Examples I

- $\displaystyle\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx = -\int \frac{\cos'(x)}{\cos(x)}\,dx = -\log(\cos(x)).$
- Consider $\int x^2 \tan(x^3)\,dx$. Put $u = x^3$, so $du = 3x^2\,dx$, so $dx = du/(3x^2)$.

$$\begin{aligned}\int x^2 \tan(x^3)\,dx &= \int x^2 \tan(u)\frac{du}{3x^2} = \frac{1}{3}\int \tan(u)\,du = -\log(\cos(u))/3 \\ &= -\log(\cos(x^3))/3\end{aligned}$$

- Consider $\int xe^{\sqrt{x}}\,dx$. Put $t = \sqrt{x}$, so $x = t^2$, so $dx = 2t\,dt$.

$$\int xe^{\sqrt{x}}\,dx = \int t^2 e^t.2t\,dt$$

Examples I

- $\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx = -\int \frac{\cos'(x)}{\cos(x)}\,dx = -\log(\cos(x))$.
- Consider $\int x^2\tan(x^3)\,dx$. Put $u = x^3$, so $du = 3x^2\,dx$, so $dx = du/(3x^2)$.

$$\int x^2\tan(x^3)\,dx = \int x^2\tan(u)\frac{du}{3x^2} = \frac{1}{3}\int \tan(u)\,du = -\log(\cos(u))/3$$
$$= -\log(\cos(x^3))/3$$

- Consider $\int xe^{\sqrt{x}}\,dx$. Put $t = \sqrt{x}$, so $x = t^2$, so $dx = 2t\,dt$.

$$\int xe^{\sqrt{x}}\,dx = \int t^2e^t.2t\,dt = 2\int t^3e^t\,dt$$

Examples I

- $\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx = -\int \frac{\cos'(x)}{\cos(x)}\,dx = -\log(\cos(x))$.
- Consider $\int x^2\tan(x^3)\,dx$. Put $u = x^3$, so $du = 3x^2\,dx$, so $dx = du/(3x^2)$.

$$\int x^2\tan(x^3)\,dx = \int x^2\tan(u)\frac{du}{3x^2} = \frac{1}{3}\int \tan(u)\,du = -\log(\cos(u))/3$$
$$= -\log(\cos(x^3))/3$$

- Consider $\int xe^{\sqrt{x}}\,dx$. Put $t = \sqrt{x}$, so $x = t^2$, so $dx = 2t\,dt$.

$$\int xe^{\sqrt{x}}\,dx = \int t^2e^t.2t\,dt = 2\int t^3e^t\,dt = 2(t^3 - 3t^2 + 6t - 6)e^t$$

Examples I

- $\displaystyle\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx = -\int \frac{\cos'(x)}{\cos(x)}\,dx = -\log(\cos(x)).$
- Consider $\int x^2\tan(x^3)\,dx$. Put $u = x^3$, so $du = 3x^2\,dx$, so $dx = du/(3x^2)$.

$$\begin{aligned}\int x^2\tan(x^3)\,dx &= \int x^2\tan(u)\frac{du}{3x^2} = \frac{1}{3}\int \tan(u)\,du = -\log(\cos(u))/3 \\ &= -\log(\cos(x^3))/3\end{aligned}$$

- Consider $\int xe^{\sqrt{x}}\,dx$. Put $t = \sqrt{x}$, so $x = t^2$, so $dx = 2t\,dt$.

$$\begin{aligned}\int xe^{\sqrt{x}}\,dx &= \int t^2e^t.2t\,dt = 2\int t^3e^t\,dt = 2(t^3 - 3t^2 + 6t - 6)e^t \\ &= (2x^{3/2} - 6x + 12x^{1/2} - 12)e^{\sqrt{x}}\end{aligned}$$

Examples II

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- $$\int (2(x^2 + 1)e^x)^2 \, dx$$

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- $$\int (2(x^2+1)e^x)^2\,dx = \int (4x^4+8x^2+4)e^{2x}\,dx$$

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- $$\int (2(x^2+1)e^x)^2\,dx = \int (4x^4+8x^2+4)e^{2x}\,dx$$
  $$= (Ax^4+Bx^3+Cx^2+Dx+E)e^{2x}$$

Examples II

- $$\int (2(x^2+1)e^x)^2\,dx = \int (4x^4+8x^2+4)e^{2x}\,dx$$
  $$= (Ax^4+Bx^3+Cx^2+Dx+E)e^{2x}$$
  $$(4x^4+8x^2+4)e^{2x} = \frac{d}{dx}((Ax^4+Bx^3+Cx^2+Dx+E)e^{2x})$$

Examples II

- $$\begin{aligned}\int (2(x^2+1)e^x)^2\,dx &= \int (4x^4+8x^2+4)e^{2x}\,dx \\ &= (Ax^4+Bx^3+Cx^2+Dx+E)e^{2x} \\ (4x^4+8x^2+4)e^{2x} &= \frac{d}{dx}((Ax^4+Bx^3+Cx^2+Dx+E)e^{2x}) \\ &= (4Ax^3+3Bx^2+2Cx+D)e^{2x} + \\ &\quad (Ax^4+Bx^3+Cx^2+Dx+E).2e^{2x}\end{aligned}$$

Examples II

- $$\begin{aligned}
\int (2(x^2+1)e^x)^2\,dx &= \int (4x^4+8x^2+4)e^{2x}\,dx \\
&= (Ax^4+Bx^3+Cx^2+Dx+E)e^{2x} \\
({\color{red}4}x^4+{\color{green}8}x^2+{\color{blue}4})e^{2x} &= \frac{d}{dx}((Ax^4+Bx^3+Cx^2+Dx+E)e^{2x}) \\
&= (4Ax^3+3Bx^2+2Cx+D)e^{2x}+ \\
&\quad (Ax^4+Bx^3+Cx^2+Dx+E).2e^{2x} \\
&= e^{2x}({\color{red}2A}x^4+(4A+2B)x^3+{\color{green}(3B+2C)}x^2+ \\
&\qquad (2C+2D)x+{\color{blue}(D+2E)})
\end{aligned}$$

Examples II

- $$\begin{aligned}\int (2(x^2+1)e^x)^2\,dx &= \int (4x^4+8x^2+4)e^{2x}\,dx \\ &= (Ax^4+Bx^3+Cx^2+Dx+E)e^{2x} \\ (4x^4+8x^2+4)e^{2x} &= \frac{d}{dx}((Ax^4+Bx^3+Cx^2+Dx+E)e^{2x}) \\ &= (4Ax^3+3Bx^2+2Cx+D)e^{2x} + \\ &\quad (Ax^4+Bx^3+Cx^2+Dx+E).2e^{2x} \\ &= e^{2x}(2Ax^4+(4A+2B)x^3+(3B+2C)x^2+ \\ &\qquad (2C+2D)x+(D+2E))\end{aligned}$$

So $4=2A$, $0=4A+2B$, $8=3B+2C$, $0=2C+2D$, $4=D+2E$

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- $$\begin{aligned}\int (2(x^2+1)e^x)^2\,dx &= \int (4x^4+8x^2+4)e^{2x}\,dx \\ &= (Ax^4+Bx^3+Cx^2+Dx+E)e^{2x} \\ (4x^4+8x^2+4)e^{2x} &= \frac{d}{dx}((Ax^4+Bx^3+Cx^2+Dx+E)e^{2x}) \\ &= (4Ax^3+3Bx^2+2Cx+D)e^{2x} + \\ &\quad (Ax^4+Bx^3+Cx^2+Dx+E).2e^{2x} \\ &= e^{2x}(2Ax^4+(4A+2B)x^3+(3B+2C)x^2+ \\ &\qquad (2C+2D)x+(D+2E))\end{aligned}$$

  So $4=2A$, $0=4A+2B$, $8=3B+2C$, $0=2C+2D$, $4=D+2E$
  So $A=2,\ B=-4,\ C=10,\ D=-10,\ E=7$

Examples II

- $$\int (2(x^2+1)e^x)^2\,dx = \int (4x^4+8x^2+4)e^{2x}\,dx$$
$$= (Ax^4+Bx^3+Cx^2+Dx+E)e^{2x}$$
$$(4x^4+8x^2+4)e^{2x} = \frac{d}{dx}((Ax^4+Bx^3+Cx^2+Dx+E)e^{2x})$$
$$= (4Ax^3+3Bx^2+2Cx+D)e^{2x} + (Ax^4+Bx^3+Cx^2+Dx+E).2e^{2x}$$
$$= e^{2x}(2Ax^4+(4A+2B)x^3+(3B+2C)x^2+(2C+2D)x+(D+2E))$$

So $4 = 2A$, $0 = 4A+2B$, $8 = 3B+2C$, $0 = 2C+2D$, $4 = D+2E$
So $A = 2,\ B = -4,\ C = 10,\ D = -10,\ E = 7$

$$\int (2(x^2+1)e^x)^2\,dx = (2x^4-4x^3+10x^2-10x+7)e^{2x}.$$

Examples III

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Examples III

- $$\int 1 + \cosh(x) + \cosh(x)^2 \, dx = \int 1 + \frac{e^x + e^{-x}}{2} + \left(\frac{e^x + e^{-x}}{2}\right)^2 \, dx$$
$$= \frac{1}{4} \int 4 + 2e^x + 2e^{-x} + e^{2x} + 2 + e^{-2x} \, dx$$

Examples III

- $$\begin{aligned}\int 1+\cosh(x)+\cosh(x)^2\,dx &= \int 1+\frac{e^x+e^{-x}}{2}+\left(\frac{e^x+e^{-x}}{2}\right)^2\,dx\\ &= \frac{1}{4}\int 4+2e^x+2e^{-x}+e^{2x}+2+e^{-2x}\,dx\\ &= \frac{1}{4}\left(6x+2e^x-2e^{-x}+\tfrac{1}{2}e^{2x}-\tfrac{1}{2}e^{-2x}\right)\end{aligned}$$

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Examples IV

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- To show that $\int \frac{dx}{\cos(x)} = \log\left(\frac{1+\sin(x)}{\cos(x)}\right)$:

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- Consider $\int 10e^{-x}\sin(x)^2\,dx = \int 5e^{-x}\,dx + \int -5e^{-x}\cos(2x)\,dx.$

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$$A = 1, \qquad B = -2$$

Examples V

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$$\int 10e^{-x}\sin(x)^2\,dx = -5e^{-x} + e^{-x}\cos(2x) - 2e^{-x}\sin(2x).$$