Formulae for MAS100

No formula sheets etc will be permitted in the exam. You should remember (or be able to derive) all the formulae listed below. You should also remember all the rules and methods in the lectures, which are not explicitly listed here.

$$\exp(x+y) = \exp(x)\exp(y) \qquad \ln(xy) = \ln(x) + \ln(y)$$
$$\exp(x-y) = \exp(x)/\exp(y) \qquad \ln(x/y) = \ln(x) - \ln(y)$$
$$\exp(0) = 1 \qquad \ln(1) = 0$$
$$\exp(-x) = 1/\exp(x) \qquad \ln(1/y) = -\ln(y)$$
$$\exp(nx) = \exp(x)^n \qquad \ln(y^n) = n\ln(y)$$
$$\exp(x) = e^x$$

$$\log_a(y) = \ln(y)/\ln(a) = \text{ the number } x \text{ such that } a^x = y\ .$$

$\log(x)$ means the same as $\ln(x)$.

$$\sinh(x) = (e^x - e^{-x})/2 \qquad \operatorname{csch}(x) = 1/\sinh(x) = 2/(e^x - e^{-x})$$
$$\cosh(x) = (e^x + e^{-x})/2 \qquad \operatorname{sech}(x) = 1/\cosh(x) = 2/(e^x + e^{-x})$$
$$\tanh(x) = \sinh(x)/\cosh(x) = (e^x - e^{-x})/(e^x + e^{-x}) \qquad \coth(x) = 1/\tanh(x) = (e^x + e^{-x})/(e^x - e^{-x}).$$

Angles are always measured in radians, not degrees.

$$\tan(\theta) = \sin(\theta)/\cos(\theta) \qquad \cot(\theta) = \cos(\theta)/\sin(\theta)$$
$$\sec(\theta) = 1/\cos(\theta) \qquad \csc(\theta) = 1/\sin(\theta).$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y) \qquad \sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$
$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y) \qquad \cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$
$$\sin(2x) = 2\sin(x)\cos(x) \qquad \cos(2x) = \cos(x)^2 - \sin(x)^2$$
$$= 2\cos(x)^2 - 1 = 1 - 2\sin(x)^2$$
$$\sin(x)^2 = \tfrac{1}{2} - \tfrac{1}{2}\cos(2x) \qquad \cos(x)^2 = \tfrac{1}{2} + \tfrac{1}{2}\cos(2x)$$

θ	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$
$\pi/2$	1	0	∞
$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$

$$\exp'(x) = \exp(x) \qquad \ln'(x) = 1/x$$
$$\sinh'(x) = \cosh(x) \qquad \operatorname{arcsinh}'(x) = (1+x^2)^{-1/2}$$
$$\cosh'(x) = \sinh(x) \qquad \operatorname{arccosh}'(x) = (x^2-1)^{-1/2}$$
$$\tanh'(x) = \operatorname{sech}(x)^2 = 1 - \tanh(x)^2 \qquad \operatorname{arctanh}'(x) = (1-x^2)^{-1}$$
$$\sin'(x) = \cos(x) \qquad \arcsin'(x) = (1-x^2)^{-1/2}$$
$$\cos'(x) = -\sin(x) \qquad \arccos'(x) = -(1-x^2)^{-1/2}$$
$$\tan'(x) = \sec(x)^2 = 1 + \tan(x)^2 \qquad \arctan'(x) = (1+x^2)^{-1}$$

$$\int \exp(x)\,dx = \exp(x) \qquad \int \ln(x)\,dx = x\ln(x) - x$$

$$\int \sin(x)\,dx = -\cos(x) \qquad \int \cos(x)\,dx = \sin(x)$$

$$\int \sin(x)^2\,dx = \frac{2x - \sin(2x)}{4} \qquad \int \cos(x)^2\,dx = \frac{2x + \sin(2x)}{4}$$

$$\int \frac{dx}{1+x^2} = \arctan(x) \qquad \int \frac{dx}{1-x^2} = \operatorname{arctanh}(x)$$

$$\int \frac{dx}{\sqrt{1+x^2}} = \operatorname{arcsinh}(x) \qquad \int \frac{dx}{\sqrt{1-x^2}} = \arcsin(x)$$

$$\int \frac{dx}{\sqrt{x^2-1}} = \operatorname{arccosh}(x) \qquad \int \tan(x)\,dx = -\ln(|\cos(x)|)$$

$$\int a^x\,dx = a^x/\ln(a)$$

$$\int x^k\,dx = x^{k+1}/(k+1)$$

$$\int (x-a)^{-1}\,dx = \ln(|x-a|)$$

$$\int (x-a)^{-k}\,dx = (x-a)^{1-k}/(1-k) \qquad \text{(for } k > 1\text{)}$$