Maths with Maple — Week 5 tutorial

Differentiation 1

Exercises 7, 8, 9 and 10 will appear on the online test. Maple will not be too much help in answering these, but you can use it if you like. You can also use Maple to check your answers to the other questions, but you should remember that you will have to do such questions by hand in the exam.

Exercise 1. Find $\frac{d}{dx}\left(\frac{x^2}{\ln(x)}\right)$.

Exercise 2. Let a, b, c and d be constants and put $y = (ax+b)/(cx+d)$. Calculate dy/dx, simplifying your answer as much as possible.

Exercise 3. Let a, b, c and d be constants. Find $\frac{d}{dx}\left(\frac{ax+bx^{-1}}{cx+dx^{-1}}\right)$.

Exercise 4. Simplify $(x^2+y^2)^{-1}\frac{dx}{dt}$, where $x = \cos(t)/(1+a\cos(t))$ and $y = \sin(t)/(1+a\cos(t))$.

Exercise 5. Calculate the derivatives of the functions $(x^2-2x+2)e^x$, $(x^3-3x^2+6x-6)e^x$ and $(x^4-4x^3+12x^2-24x+24)e^x$. What is the next thing in this sequence?

Exercise 6. Suppose that $y = (pq)/(rs)$, where p, q, r and s all depend on x. Simplify y'/y. (You should write your answer as a sum of four terms, not as a single fraction.) Hence find y'/y when $y = x(x+3)/((x+1)(x+2))$, simplifying your answer as much as possible.

Exercise 7. Recall that a function $f(x)$ is *even* if $f(-x) = f(x)$ for all x, and *odd* if $f(-x) = -f(x)$. Note that most functions are neither even nor odd.

(a) By drawing pictures, convince yourself that if $f(x)$ is odd then $f'(x)$ is even.
(b) By drawing pictures, convince yourself that if $f'(x)$ is odd then $f(x)$ is even.
(c) Find a function $f(x)$ that is neither even nor odd. Try to make your answer as simple as possible.
(d) Find a function $f(x)$ such that $f'(x)$ is even but $f(x)$ is not odd. Try to make your answer as simple as possible.

Exercise 8. Find a function $f(x)$ such that $f'(-1) = f'(0) = f'(1) = 0$ and $f(0) < f(1)$. Try to make your answer as simple as possible.

Exercise 9. Find a function $f(x)$ such that

(a) $f(x)$ is defined for all x (without any division by zero, square roots of negative numbers, or other horrors.)
(b) $f'(x) < 0$ for all x (so $f(x)$ is continuously decreasing)
(c) $f(x) > 0$ for all x (so although $f(x)$ is decreasing, it never reaches zero.)

Exercise 10. Consider the function $f(x) = ax^2+bx+c$, where $a,b,c > 0$. Suppose that there is a point x_0 where $f(x_0) = f'(x_0) = 0$. Give formulae for x_0 and c in terms of a and b. (You might like to draw some pictures first.)

Exercise 11. Let c be a positive constant, and put $g(v) = (1-v^2/c^2)^{-1/2}$. Calculate $g'(v)$.

Exercise 12. Calculate $\frac{d}{dx}\left(e^{-a^2x^2}\sin(\omega x)\right)$, where a and ω are constants.

Exercise 13. Let p and q be nonzero constants, and put $y = (x^p - x^q)^{1/pq}$. Simplify $x(x^p-x^q)\frac{dy}{dx}$.

Exercise 14. Let a, b and n be constants. Find $f'(x)$, where $f(x) = \left(\frac{x-a}{x-b}\right)^n$.

Exercise 15. Put $y = a_1x + a_2x^2 + a_3x^3 + a_4x^4$.

(a) Find $x\frac{dy}{dx}$.
(b) Find $x\frac{d}{dx}\left(x\frac{dy}{dx}\right)$.
(c) Find $x\frac{d}{dx}\left(x\frac{d}{dx}\left(x\frac{dy}{dx}\right)\right)$.
(d) What is the general rule?

Exercise 16. Given that y is a function of x, simplify the following expressions:

(a) $e^{-x}\frac{d}{dx}(e^xy)$
(b) $e^{-x}\frac{d^2}{dx^2}(e^xy)$
(c) $e^{-x}\frac{d^3}{dx^3}(e^xy)$

Can you guess the general rule? Can you prove it?

Exercise 17. Put $f(x) = x/\sqrt{1+x^2}$. Simplify $\sqrt{1+x^2}f'(x)$, and hence find a constant c such that $f'(x) = (1+x^2)^c$.

Exercise 18. Calculate the derivatives of the following functions:

$$\begin{aligned} f(x) &= x + x^2/2 + x^3/3 + x^4/4 + x^5/5 \\ g(x) &= \frac{1 + x^2 + x^4}{x + x^3} \\ h(x) &= \tan(x)^7 \\ k(x) &= \arcsin(x) \\ m(x) &= \ln(\cos(x)). \end{aligned}$$

(In the case of $k(x)$ you should give an argument starting with your knowledge of $\sin'(x)$, rather than just quoting the answer from tables or your memory.)