

# Integration 1

**Exercise 1.** Find  $\int_0^5 (x^2 + 15)^2 dx$

**Exercise 2.** Let  $\lambda$  be a positive constant. Then there are constants  $m$  and  $b$  such that for all  $a$  we have

$$\log \left[ \int_a^{a+1} e^{\lambda x} dx \right] = ma + b.$$

Find formulae for  $m$  and  $b$  in terms of  $\lambda$ .

**Exercise 3.** Find an integer  $n > 0$  such that  $\int_{-\pi}^{\pi} \cos(x/n) dx = n$ .

**Exercise 4.** Show that  $\int \frac{dx}{\sin(2x)} = \frac{1}{2} \log(\tan(x))$ , and thus calculate  $\exp \left( \int_{\pi/6}^{\pi/3} \frac{dx}{\sin(2x)} \right)$ .

(Note that the question does **not** ask you to find  $\int \frac{dx}{\sin(2x)}$  from scratch; it suggests an answer, and asks you to show that it is correct.)

**Exercise 5.** Show that  $\int \tanh(x) dx = \log(e^{2x} + 1) - x$ .

**Exercise 6.** For any function  $f(x)$ , evaluate  $\int \frac{f'(x)}{f(x)} dx$ .

**Exercise 7.** Find  $\frac{d}{dx}(e^{3x}(A \sin(4x) + B \cos(4x)))$ . Find  $A$  and  $B$  such that this derivative is equal to  $e^{3x} \sin(4x)$ . Hence find  $\int_0^{2\pi} e^{3x} \sin(4x) dx$ .

**Exercise 8.** The indefinite integral  $\int \ln(x)^2 - \ln(x) + 1 dx$  has the form  $(u \ln(x)^2 + v \ln(x) + w)x$  for some constants  $u$ ,  $v$  and  $w$ . Find these constants.

**Exercise 9.** Find  $\frac{d}{dx}(x^a \ln(x)^b)$ . By choosing  $a$  and  $b$  suitably, find the integral  $\int \ln(x)^{-2} - \ln(x)^{-1} dx$ .

**Exercise 10.** Find  $\int x e^x \cos(x) dx$ .

**Exercise 11.** Find  $\int f(x) dx$ , where  $f(x) = x^3 e^{3x}$ . You may assume that the answer has the form  $F(x) = (Ax^3 + Bx^2 + Cx + D)e^{3x}$  for some constants  $A, B, C$  and  $D$ .

**Exercise 12.** Let  $p(x)$  be a polynomial of degree 3, say

$$p(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3.$$

- Show that  $p'''(x) = 0$ .
- Put  $q(x) = p(x) + p'(x) + p''(x) + p'''(x)$ . Show that  $q(x) - q'(x) = p(x)$ .
- Deduce that  $\int p(x)e^{-x} dx = -q(x)e^{-x}$ .
- How should this be adjusted for polynomials of degree 4, or of degree  $n$  for arbitrary  $n$ ?

**Exercise 13.** You may assume that  $\int x^2 \log(x)^2 dx = x^3(a \log(x)^2 + b \log(x) + c)$

for some constants  $a$ ,  $b$  and  $c$ . Find these constants, and thus evaluate  $\int_1^e x^2 \log(x)^2 dx$ .

**Exercise 14.** Use the general formula

$$\int \frac{dx}{\sqrt{ax^2 + bx + c}} = \ln(2\sqrt{a^2x^2 + abx + ac} + 2ax + b)/\sqrt{a},$$

to find the integral

$$\int \frac{dx}{\sqrt{4x^2 + 5x + 6}}.$$