Integration 2

Exercise 1. Expand out $\cosh(x)^3$ in terms of exponentials, and thus evaluate $\int \cosh(x)^3 \, dx$. Give your answer in terms of exponentials, and then rewrite it in terms of sinh and cosh.

Exercise 2. Find $\int \cosh(x)^2 \, dx$

Exercise 3. Find $\int 2 \sinh(3x) - (2 \sinh(x))^3 \, dx$.

Exercise 4. Recall that
$$\sin(2x) = 2 \sin(x) \cos(x)$$
$$\cos(2x) = \cos(x)^2 - \sin(x)^2 = 2 \cos(x)^2 - 1 = 1 - 2 \sin(x)^2.$$

Use these to convert the following integrals to a form in which they can easily be evaluated, and then evaluate them.

(a) $\displaystyle\int \sin(x)^2 \, dx$ **(b)** $\displaystyle\int \sin(x) \cos(x) \, dx$ **(c)** $\displaystyle\int \sin(x)^2 \cos(x)^2 \, dx$ **(d)** $\displaystyle\int \sin(x) \cos(x)^3 - \sin(x)^3 \cos(x) \, dx$

Now check by differentiating that $\int \sin(x) \cos(x) \, dx = \sin(x)^2/2$. Is this consistent with your answer to (b)?

Exercise 5.   (a) Let p be an integer. Find $\int_0^{2\pi} \cos(p\theta) \, d\theta$. (Note that the case $p = 0$ must be considered separately.)
(b) Let n and m be positive integers. Using the relation $\cos(\theta) = (e^{i\theta} + e^{-i\theta})/2$, show that
$$\cos(n\theta) \cos(m\theta) = \tfrac{1}{2} \cos((n+m)\theta) + \tfrac{1}{2} \cos((n-m)\theta).$$

(c) Using (a) and (b), show that $\int_0^{2\pi} \cos(n\theta) \cos(m\theta) = 0$ whenever $n, m > 0$ and $n \neq m$. What is the value of the integral when $n = m$?

Exercise 6. Find $\int e^{-3x} \cos(4x) \, dx$

Exercise 7. Find $\int \sqrt{2} e^{-x/\sqrt{2}} \sin(x/\sqrt{2}) \, dx$

Exercise 8. Find $\int e^{-x} \sin(x)^2 \, dx$. (The first step is to rewrite $\sin(x)^2$, just as you would if you were doing $\int \sin(x)^2 \, dx$.)

Exercise 9. The general form of $\int xe^{-x} \cos(x)$ is $e^{-x}((Ax + B) \cos(x) + (Cx + D) \sin(x))$. In the same way, write down the general form of each of the following integrals:

(a) $\displaystyle\int x^2 e^{3x} \sin(4x) \, dx$ **(b)** $\displaystyle\int x^6 e^{x/6} \, dx$ **(c)** $\displaystyle\int (1 + x + x^2) \sin(x) \, dx$ **(d)** $\displaystyle\int (1 - x) e^x (\sin(x) - \cos(x)) \, dx$

(You need not find the coefficients, just write down the general form.)

Exercise 10. Find $\int 8x \sin(x) \cos(x) \, dx$

Exercise 11. Find $\int x^2 e^x \, dx$

Exercise 12. Find $\int (4x^2 + 2x + 1) e^{2x} \, dx$.