

## Integration 3

**Exercise 1.** Use integration by parts to find  $\int x \cos(\omega x) dx$

**Exercise 2.** Use integration by parts twice to find  $\int x^2 e^{3x} dx$ .

**Exercise 3.** Find  $\int x^n \ln(x) dx$ .

**Exercise 4.** Put  $I = \int e^x \sin(x) dx$  and  $J = \int e^x \cos(x) dx$ . Try to integrate  $I$  by parts; you should obtain an equation relating  $I$  and  $J$ . Now try to integrate  $J$  by parts; you should obtain another equation relating  $I$  and  $J$ . Solve these two equations simultaneously to find  $I$  and  $J$ .

**Exercise 5.** By making suitable substitutions, find the following integrals:

$$(a) \int x e^{-x^2} dx \quad (b) \int x^4 \sin(x^5) dx \quad (c) \int (3x^2 - 1) \cos(x^3 - x) dx \quad (d) \int (1 - x^{-2}) \ln(x + x^{-1}) dx$$

**Exercise 6.** Evaluate  $\int_0^{\pi/4} \tan(x) dx$  by the substitution  $u = \cos(x)$  (remembering that  $\tan(x) = \sin(x)/\cos(x)$ ).

**Exercise 7.** By putting  $u = \ln(x)$ , find  $\int_{1/e}^e \frac{(1 + \ln(x))^2}{x} dx$ .

**Exercise 8.** By substituting  $u = x^n$ , find  $\int \frac{dx}{x\sqrt{x^{-2n} - 1}}$ .

**Exercise 9.** Find  $\int x^{-2} e^{-2/x} dx$ .

**Exercise 10.** Find the integral  $\int \frac{(1+x)^5}{(1-x)^7} dx$ , by substituting  $u = (1+x)/(1-x)$ .

**Exercise 11.** Find the integral  $\int e^{\epsilon^x} e^x dx$ .

**Exercise 12.** Find  $\int x^{n-1} (1+x^n)^{m-1} dx$  (where  $n$  and  $m$  are constants with  $n, m > 0$ ).

**Exercise 13.** Find the integral  $\int_0^1 \frac{x dx}{\sqrt{1-x^4}}$ , by substituting  $u = x^2$ .

**Exercise 14.** By making a suitable substitution, find  $\int \sin(x) \ln(\cos(x)) dx$ .

**Exercise 15.** By substituting  $u = x^{n+1}$ , find  $\int x^n \ln(x) dx$ .

**Exercise 16.** Find  $\int_0^\infty x e^{-4x^2} dx$ .