Maths with Maple — Week 11 tutorial

Revision

The problems on this sheet are typical questions from the second half of the exam. (The first half of the exam covers Maple.)

Exercise 1. Show that $\dfrac{1}{\tanh(x)} - \tanh(x) = \dfrac{2}{\sinh(2x)}$.

Exercise 2. Show that $\dfrac{1+\tanh(x)^2}{1-\tanh(x)^2} = \cosh(2x)$.

Exercise 3. Find $\int \sinh(x)^3\,dx$, expressing your answer in terms of hyperbolic functions.

Exercise 4. Let a, b and n be constants. Find $f'(x)$, where $f(x) = \left(\dfrac{x-a}{x-b}\right)^n$.

Exercise 5. Find $\dfrac{d}{dx}(\log(x^2+x^{-2}))$. Simplify your answer as much as possible.

Exercise 6. Find $\dfrac{d}{dx}\left(\dfrac{x^4+x^2+1}{x^3-x}\right)$.

Exercise 7. Find dy/dx in terms of x and y, where x and y are related by the equation $e^{-x^2-xy-y^2}\sin(x) = a$.

Exercise 8. Find dv/du, where u and v are related by the equation $uv^2 + u^2v^3 = 1$.

Exercise 9. Find dy/dx in terms of t, where $x = t - \sin(t)$ and $y = 1 - \cos(t)$.

Exercise 10. By making a suitable substitution, find $\int \sin(x)\log(\cos(x))\,dx$.

Exercise 11. Find $\displaystyle\int_0^1 x\sin(\pi x)\,dx$.

Exercise 12. Find $\int (x^3+x^2+x+1)e^{-x}\,dx$.

Exercise 13. You may assume that

$$\int \cos(x)^{-4}\,dx = \tan(x)\left(a + b\cos(x)^{-2}\right)$$

for some constants a and b. Find these constants.
[Hint: you will need the identities $\tan(x) = \sin(x)/\cos(x)$ and $\sin(x)^2 = 1 - \cos(x)^2$.]