MAS201 PROBLEM SHEET 1

Please hand in your answers for exercises 5 and 10 (together with two more exercises that will appear on Problem Sheet 2) in the tutorial in week 3.

For the remaining exercises, please enter your answers in the online test system (which can be reached from the course home page at `http://shef.ac.uk/nps/courses/MAS201`). The system will check whether your answers are correct; if not, it will often be able to give feedback about what went wrong.

Lecture 1

Exercise 1. Calculate AB, where

$$A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 3 & 2 & 0 & 0 \\ 3 & 6 & 2 & 0 \\ 3 & 6 & 3 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 2 & 2 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Exercise 2. Consider the following matrices:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 10 \\ 100 & 1000 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \\ 11 & 0 \\ 111 & 0 \end{bmatrix}.$$

For each of the following products, either evaluate the product or explain why it is undefined:

$$A^2 \quad AB \quad AC \quad BA \quad B^2 \quad BC \quad CA \quad CB \quad C^2$$

Exercise 3. Find examples as follows.

(a) Matrices A and B such that AB is defined but BA is not.
(b) Matrices C and D such that CD and DC are both defined but have different sizes.
(c) Matrices E and F such that EF and FE are both defined and have the same size but are not equal.
(d) Matrices G and H such that GH and HG are both defined and have the same size and are equal.

Exercise 4. Find a nonzero matrix A such that A^2 is defined and is zero.

Exercise 5. The *trace* of a square matrix is the sum of the diagonal entries. Show that if $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$ then the trace of $AB - BA$ is zero.

Lecture 2

Exercise 6. Which of the following matrices are in reduced row-echelon form?

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & 1 & 0 & 2 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad E = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Exercise 7. Give an example of a 4×7 matrix in RREF with pivots in columns 2, 5 and 7 (and no other columns) and with precisely six nonzero entries.

Exercise 8. Use the augmented matrix method to solve the following system of linear equations, or prove that there is no solution.

$$\begin{aligned} 10a &= 10b + c \\ 10c + b &= 10a - 9 \\ a + 100c &= 100b + 11. \end{aligned}$$

Exercise 9. Use the augmented matrix method to solve the following system of linear equations, or prove that there is no solution.

$$\begin{aligned} 2w - x - y - 2z &= 1 \\ 3w - 2x - 2y - 3z &= -1 \\ 5w - 3x - 3y - 5z &= 0. \end{aligned}$$

Exercise 10. Use the augmented matrix method to solve the following system of linear equations, or prove that there is no solution.

$$\begin{aligned} p + q + r &= 30 \\ p + q - r &= 16 \\ p - q + r &= 24 \\ p - q - r &= 11 \end{aligned}$$