MAS201 PROBLEM SHEET 3

Please hand in your answers for exercises 1 and 6 (together with two more exercises that will appear on Problem Sheet 4) in the tutorial in week 5. For the remaining exercises, please enter your answers in the online test system (which can be reached from the course home page at `http://shef.ac.uk/nps/courses/MAS201`). The system will check whether your answers are correct; if not, it will often be able to give feedback about what went wrong.

Lecture 5

Exercise 1. You should justify your answers to the following questions.

(a) Is the list $a_1 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}, a_2 = \begin{bmatrix} 2 \\ 7 \end{bmatrix}, a_3 = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$ a basis for $\mathbb{R}^2$?

(b) Is the list $b_1 = \begin{bmatrix} 9 \\ 8 \\ 7 \end{bmatrix}, b_2 = \begin{bmatrix} 8 \\ 7 \\ 6 \end{bmatrix}, b_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ a basis for $\mathbb{R}^3$?

(c) Is the list $c_1 = \begin{bmatrix} 1 \\ 8 \\ 5 \\ 4 \end{bmatrix}, c_2 = \begin{bmatrix} 7 \\ 3 \\ 9 \\ 5 \end{bmatrix}, c_3 = \begin{bmatrix} 5 \\ 1 \\ 9 \\ 9 \end{bmatrix}$ a basis for $\mathbb{R}^4$?

Exercise 2. Consider the list

$$a_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}, \quad a_2 = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 4 \end{bmatrix}, \quad a_3 = \begin{bmatrix} 1 \\ 4 \\ 5 \\ 6 \end{bmatrix}, \quad a_4 = \begin{bmatrix} 7 \\ 8 \\ 9 \\ 10 \end{bmatrix}.$$

Is this a basis for $\mathbb{R}^4$?

Exercise 3. Put $u_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $u_2 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$. Find a vector u_3 such that the list u_1, u_2, u_3 is a basis for $\mathbb{R}^3$.

Exercise 4. Suppose that the list a_1, a_2, a_3, a_4, a_5 is a basis for $\mathbb{R}^5$. Show that the list a_1, a_3, a_5 is linearly independent.

Lecture 6

Exercise 5. Find the inverse of the matrix

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

Exercise 6. Consider the matrix

$$A_0 = \begin{bmatrix} 0 & 10 & 100 & -1 & 10 \\ 0 & 11 & 110 & -1 & 21 \\ 0 & -1 & -10 & 0 & -11 \end{bmatrix}$$

(a) Find a row reduction

$$A_0 \to A_1 \to A_2 \to A_3 \to A_4 \to A_5 \to A_6$$

where each step uses only a single row-operation and A_6 is in RREF.

(b) Find elementary matrices $U_1, \dots, U_6$ such that $A_i = U_i A_{i-1}$.

(c) Hence find an invertible matrix U such that $A_6 = U A_0$. (Be careful about the order of multiplication.)

Exercise 7. Which of the following matrices are invertible? Justify your answers.

$$A = \begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 3 & 3 & 2 \\ 2 & 3 & 3 & 2 \\ 1 & 2 & 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 2 & 2 & 2 \\ 1 & 2 & 2 & 2 \\ 1 & 1 & 2 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 1 & 1 & 1 \\ 2 & 2 & 1 & 1 \\ 2 & 2 & 2 & 1 \\ 2 & 2 & 2 & 2 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 5 & 6 \\ 0 & 0 & 7 & 8 \\ 0 & 0 & 0 & 9 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 1 \\ 10 & 11 \\ 100 & 111 \\ 1000 & 1111 \end{bmatrix}$$

Exercise 8. Find the inverse of the following matrix, either by creative experimentation or by row-reduction.

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & a & b \\ 0 & 1 & c & d \end{bmatrix}$$