MAS201 PROBLEM SHEET 4

Please hand in your answers for exercises 2 and 9 (together with the exercises mentioned on Problem Sheet 3) in the tutorial in week 5. For the remaining exercises, please enter your answers in the online test system (which can be reached from the course home page at `http://shef.ac.uk/nps/courses/MAS201`). The system will check whether your answers are correct; if not, it will often be able to give feedback about what went wrong.

Lecture 7

Exercise 1. Calculate the determinant of the matrix

$$A = \begin{bmatrix} a & 0 & b & c \\ d & 0 & 0 & 0 \\ e & f & g & h \\ i & 0 & 0 & j \end{bmatrix}$$

Exercise 2. Consider the matrix

$$A = \begin{bmatrix} a & b & c & d \\ e & 0 & 0 & f \\ g & 0 & 0 & h \\ i & j & k & l \end{bmatrix}.$$

Prove that $\det(A) = \det \begin{bmatrix} e & f \\ g & h \end{bmatrix} \det \begin{bmatrix} b & c \\ j & k \end{bmatrix}$. (You can reduce the work involved if you choose carefully how to expand the determinant.)

Exercise 3. Calculate the determinant of the matrix

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}.$$

(The easiest method is to start with some carefully chosen row operations as in Method 12.9.)

Exercise 4. Find the adjugate, determinant and inverse of the matrix $C = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$.

(Note that the intermediate calculations that you need for $\det(C)$ are a subset of those that you need for $\operatorname{adj}(C)$. Try not to repeat work unnecessarily.)

Exercise 5. Find the adjugate, determinant and inverse of the matrix $H = \begin{bmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{bmatrix}$.

(Note again that the intermediate calculations that you need for $\det(H)$ are a subset of those that you need for $\operatorname{adj}(H)$.)

Lecture 8

Exercise 6. Find the characteristic polynomial of the matrix

$$A = \begin{bmatrix} 0 & 0 & 0 & -d \\ 1 & 0 & 0 & -c \\ 0 & 1 & 0 & -b \\ 0 & 0 & 1 & -a \end{bmatrix}$$

Exercise 7. Find the characteristic polynomial, eigenvalues and all the corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 4 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$$

Exercise 8. Find the characteristic polynomial, eigenvalues and all the corresponding eigenvectors of the matrix

$$B = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & -1 \end{bmatrix}$$

Exercise 9. Show, directly from the definition of eigenvalue, that 0 is an eigenvalue of the matrix

$$N = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Show, also directly from the definition of eigenvalue, that an arbitrary non-zero number k is not an eigenvalue of N. Find all the eigenvectors of N.

Exercise 10. Find the characteristic polynomial, eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 3 & -5 & 5 \\ 2 & -4 & 5 \\ 2 & -2 & 3 \end{bmatrix}.$$

Exercise 11. A be an $n \times n$ matrix, and let $\lambda_1, \ldots, \lambda_h$ be h distinct eigenvalues of A. For each $i = 1, \ldots, h$, let the vectors $v_{i,1}, \ldots, v_{i,t_i}$ be linearly independent eigenvectors of A all corresponding to the eigenvalue λ_i. We collect these lists together into a single list

$$v_{1,1}, \ldots, v_{1,t_1}, v_{2,1}, \ldots, v_{2,t_2}, \ldots, v_{h,1}, \ldots, v_{h,t_h}.$$

Prove (as was stated in lectures) that this list is linearly independent.