MAS201 PROBLEM SHEET 8

Please hand in your answers for exercises 5 and 8 (together with the exercises mentioned on Problem Sheet 7) in the tutorial in week 10. For the remaining exercises, please enter your answers in the online test system (which can be reached from the course home page at `http://shef.ac.uk/nps/courses/MAS201`).

Lecture 15

Exercise 1. Let V be the set of vectors of the form

$$v = \begin{bmatrix} 2p-q & q+r & 3p & r \end{bmatrix}^T$$

(where p, q, and r are arbitrary real numbers). Find a list of vectors whose span is V.

Exercise 2. Put

$$A = \begin{bmatrix} 1 & 6 & 8 \\ 7 & 2 & 3 \end{bmatrix}$$

and $V = \{v \in \mathbb{R}^3 \mid Av = 0\}$. Find a list of vectors whose annihilator is V.

Exercise 3. Put

$$a_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \qquad a_2 = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} \qquad u = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \qquad v = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

(a) Does u lie in $\operatorname{ann}(a_1, a_2)$?
(b) Does v lie in $\operatorname{ann}(a_1, a_2)$?
(c) Does u lie in $\operatorname{span}(a_1, a_2)$?
(d) Does v lie in $\operatorname{span}(a_1, a_2)$?

Exercise 4. Put

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} \qquad b_1 = \begin{bmatrix} 1 \\ 11 \\ 1 \end{bmatrix} \quad b_2 = \begin{bmatrix} 1 \\ 12 \\ 1 \end{bmatrix} \quad b_3 = \begin{bmatrix} 1 \\ 13 \\ 1 \end{bmatrix}$$

For each of the following subspaces, give an example of a nonzero vector that lies in the subspace, and an example of a nonzero vector that does not lie in the subspace.

$$\begin{aligned} V_0 &= \operatorname{span}(b_1, b_2, b_3) \\ V_1 &= \operatorname{ann}(b_1, b_2, b_3) \\ V_2 &= \operatorname{img}(A) \\ V_3 &= \ker(A). \end{aligned}$$

Exercise 5. Put

$$a_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \end{bmatrix} \qquad a_2 = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 1 \end{bmatrix} \qquad b_1 = \begin{bmatrix} 3 \\ -3 \\ 4 \\ -4 \end{bmatrix} \qquad b_2 = \begin{bmatrix} 4 \\ -4 \\ 3 \\ -3 \end{bmatrix}.$$

Show that $\operatorname{span}(a_1, a_2) \subseteq \operatorname{ann}(b_1, b_2)$.

Exercise 6. Put $V = \operatorname{span}(v_1, v_2, v_3)$, where

$$\begin{aligned} v_1 &= \begin{bmatrix} 0 & 2 & 6 & 10 & 1 & 0 \end{bmatrix}^T \\ v_2 &= \begin{bmatrix} 0 & 1 & 3 & 5 & 1 & -3 \end{bmatrix}^T \\ v_3 &= \begin{bmatrix} 0 & 3 & 9 & 15 & 1 & 3 \end{bmatrix}^T. \end{aligned}$$

(a) What is the dimension of V?
(b) What is the canonical basis for V?
(c) What is the set $J(V)$ of jumps for V?

Exercise 7. Let V be the set of all vectors of the form

$$v = \begin{bmatrix} p+q & p+2q & p+r & p+3r \end{bmatrix}^T.$$

You may assume that this is a subspace. Find a list of vectors that spans V, and then find the canonical basis for V.

Exercise 8. Put $V = \operatorname{span}(e_1 - e_2, e_2 - e_3, \dots, e_{n-1} - e_n) \subseteq \mathbb{R}^n$, where e_i is the i'th standard basis vector for $\mathbb{R}^n$.
(a) What is the dimension of V?
(b) What is the canonical basis for V?
(c) What is the set $J(V)$ of jumps for V?
(You can start by doing the case $n = 5$ by row-reduction if you like, but ideally you should give an answer for the general case, together with a more abstract proof that your answer is correct.)

Exercise 9. Put $V = \operatorname{ann}(a_1, a_2, a_3) \subseteq \mathbb{R}^6$, where

$$\begin{aligned} a_1 &= \begin{bmatrix} 1 & 1 & 2 & 3 & 3 & 2 \end{bmatrix}^T \\ a_2 &= \begin{bmatrix} 3 & 3 & 2 & 1 & 1 & 2 \end{bmatrix}^T \\ a_3 &= \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}^T. \end{aligned}$$

Find the canonical basis for V.

Exercise 10. Put

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 3 & 3 & 1 \\ 1 & 4 & 4 & 1 \end{bmatrix}.$$

Find the canonical basis for $\operatorname{img}(A)$, and the canonical basis for $\ker(A)$.