**Data provided: Formula Sheet**

**AMA243**

The University Of Sheffield.

**SCHOOL OF MATHEMATICS AND STATISTICS** **Spring Semester 2008–2009**

# Mathematics IV (Electrical)

2 **Hours**

*Attempt all FOUR questions.*

**1** (i) Find and classify the stationary points of the function

$$F(x, y) = x^2 + y^2 + 2x^2y - 3y.$$

***(11 marks)***

(ii) Use the method of Lagrange multipliers to find the maximum and minimum values attained by the function

$$f(x, y, z) = (x + 1)^2 + (2y + 1)^2 + (3z + 1)^2$$

subject to the constraint

$$x^2 + 4y^2 + 9z^2 = 12.$$

***(14 marks)***

**2** (i) Sketch the region over which the integral $I$ is defined, where

$$I = \int_0^1 \int_y^1 4x^6 e^{x^3 y}\, dx\, dy.$$

***(3 marks)***

Evaluate $I$ by changing the order of integration. ***(8 marks)***

(ii) Let $R$ be the region consisting of all points $(x, y)$ such that $x \geq 0$, $y \geq 0$ and $1 \leq x^2 + y^2 \leq 4$. For which values of the polar co-ordinates $r$ and $\theta$ is the point $(r, \theta)$ in the region $R$? ***(3 marks)***

Evaluate the integral

$$\iint_R (1 + x)\sqrt{x^2 + y^2}\, dA.$$

***(11 marks)***

**3** A vector field $\boldsymbol{A} = \boldsymbol{A}(x, y, z)$ is given by

$$\boldsymbol{A} = \left(8xz - z^2\right)\boldsymbol{i} + \left(3y^2\right)\boldsymbol{j} + \left(ax^2 - 2xz\right)\boldsymbol{k}$$

for some scalar $a$.

Calculate div $\boldsymbol{A}$ and curl $\boldsymbol{A}$. ***(5 marks)***

By evaluating both sides, verify that

$$\nabla^2\boldsymbol{A} = \mathrm{grad}\,\mathrm{div}\,\boldsymbol{A} - \mathrm{curl}\;\mathrm{curl}\,\boldsymbol{A}.$$

***(10 marks)***

Find the value of $a$ for which curl $\boldsymbol{A} = 0$ and find a scalar field $\phi = \phi(x, y, z)$ such that, for this value of $a$,

$$\boldsymbol{A} = \mathrm{grad}\,\phi.$$

***(10 marks)***

**4** Let $S$ be the surface consisting of the hemisphere $S_1$ given, in spherical coordinates, by $r = 1$, $0 \leq \phi \leq 2\pi$, $0 \leq \theta \leq \frac{\pi}{2}$ together with the disc $S_2$ given by $0 \leq r \leq 1$, $0 \leq \phi \leq 2\pi$, $\theta = \frac{\pi}{2}$. Let $V$ be the hemispherical volume enclosed by $S$ and let $\boldsymbol{E}$ be the vector field

$$\boldsymbol{E} = x\boldsymbol{i} + y\boldsymbol{j} + (1 - z)\,\boldsymbol{k}.$$

(i) Evaluate the surface integral

$$\iint_S \boldsymbol{E.dS}.$$

***(15 marks)***

(ii) Evaluate

$$\iiint_V \mathrm{div}\,\boldsymbol{E}\,dV$$

and verify that

$$\iiint_V \mathrm{div}\,\boldsymbol{E}\,dV = \iint_S \boldsymbol{E.dS}.$$

***(10 marks)***

**End of Question Paper**

**Formula Sheet for AMA243 Trigonometry**

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$a\cos\theta + b\sin\theta = R\cos(\theta - \alpha) \text{ where } R = \sqrt{a^2+b^2} \text{ and } \cos\alpha = \frac{a}{R},\ \sin\alpha = \frac{b}{R}$$

$$\cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)$$

$$\cos^3\theta = \frac{1}{4}(3\cos\theta + \cos 3\theta)$$

$$\cos^4\theta = \frac{1}{8}(3 + 4\cos 2\theta + \cos 4\theta)$$

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\sin^3\theta = \frac{1}{4}(3\sin\theta - \sin 3\theta)$$

$$\sin^4\theta = \frac{1}{8}(3 - 4\cos 2\theta + \cos 4\theta)$$