Divergence and curl

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Examples of div and curl

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Consider a vector field

$$\mathbf{u} = \begin{bmatrix} ax + by + cz \\ dx + ey + fz \\ gx + hy + iz \end{bmatrix}$$

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$$\begin{aligned} \operatorname{div}(\mathbf{u}) &= \frac{\partial}{\partial x}(ax + by + cz) + \frac{\partial}{\partial y}(dx + ey + fz) + \frac{\partial}{\partial z}(gx + hy + iz) \\ &= a + e + i = \operatorname{trace} \begin{bmatrix} \color{red}{a} & b & c \\ d & \color{red}{e} & f \\ g & h & \color{red}{i} \end{bmatrix} \\ \operatorname{curl}(\mathbf{u}) &= \det \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ ax + by + cz & dx + ey + fz & gx + hy + iz \end{bmatrix} \\ &= (h - f,\ c - g,\ d - b) \end{aligned}$$

Consider a vector field

$$\mathbf{u} = \begin{bmatrix} ax+by+cz \\ dx+ey+fz \\ gx+hy+iz \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Then

$$\begin{aligned} \operatorname{div}(\mathbf{u}) &= \frac{\partial}{\partial x}(ax+by+cz) + \frac{\partial}{\partial y}(dx+ey+fz) + \frac{\partial}{\partial z}(gx+hy+iz) \\ &= a+e+i = \operatorname{trace}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \\ \operatorname{curl}(\mathbf{u}) &= \det\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ ax+by+cz & dx+ey+fz & gx+hy+iz \end{bmatrix} \\ &= (h-f,\ c-g,\ d-b) \end{aligned}$$

So $\operatorname{curl}(\mathbf{u}) = 0$ if the matrix $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ is symmetric.

grad, div and curl in two dimensions

(a) For a scalar field f in two dimensions, $\mathrm{grad}(f) = \nabla(f) = (f_x, f_y)$ (a vector field).

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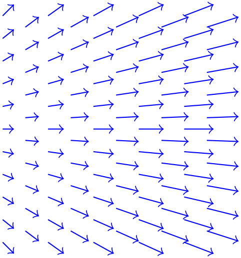
(c) For a vector field $\mathbf{u} = (p, q)$ in two dimensions,

$$\text{curl}(\mathbf{u}) = \det \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ p & q \end{bmatrix} = q_x - p_y$$

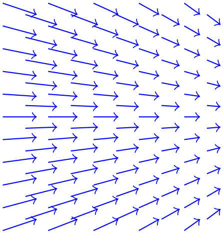
(a *scalar* field, not a vector field as in three dimensions).

Geometric interpretation of div($\mathbf{u}$)

It works out that the divergence div($\mathbf{u}$) $= \nabla.\mathbf{u}$ is positive when the vectors $\mathbf{u}$ are spreading out, and negative when they are coming together.



diverging: $\nabla.\mathbf{u} > 0$

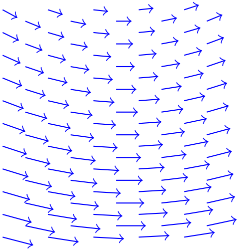


converging: $\nabla.\mathbf{u} < 0$

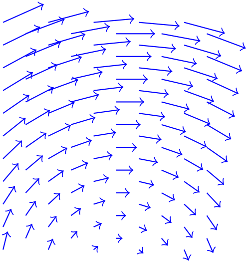
For the velocity field of an incompressible fluid we will have $\nabla.\mathbf{u} = 0$.

Geometric interpretation of curl($\mathbf{u}$)

In two dimensions, it works out that curl($\mathbf{u}$) > 0 in regions where the field is curling anticlockwise, and curl($\mathbf{u}$) < 0 in regions where it is curling clockwise, and the absolute value of curl($\mathbf{u}$) is determined by the strength of the curling.



curl($\mathbf{u}$) > 0, smaller



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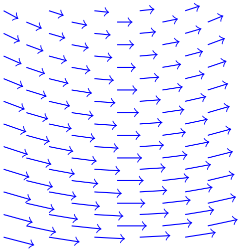
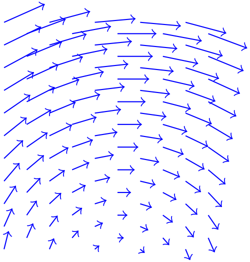


curl($\mathbf{u}$) > 0, smaller



curl($\mathbf{u}$) < 0, larger

In three dimensions, the field $\mathbf{u}$ can curl around any axis. In this context, curl($\mathbf{u}$) is also a vector field, and it will point along the axis of the curling.

Maxwell's equations

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- The electric field $\mathbf{E}$, which is a vector field.

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- The electric field **E**, which is a vector field.
- The magnetic field **B**, which is another vector field.

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- The current density $\mathbf{J}$, which is also a vector field.
- The charge density ρ, which is a scalar field.

Maxwell's equations

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- The electric field **E**, which is a vector field.
- The magnetic field **B**, which is another vector field.
- The current density **J**, which is also a vector field.
- The charge density ρ, which is a scalar field.
- Two constants: $\epsilon_0 \simeq 8.854 \times 10^{-12} F/m^2$ and $\mu_0 \simeq 1.257 \times 10^{-6} Hm^{-1}$.

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The quantities $\mathbf{E}$, $\mathbf{B}$, $\mathbf{J}$ and ρ may also depend on time; we write $\dot{\mathbf{E}}$ for $\partial \mathbf{E}/\partial t$ and so on. The various fields are related by the following equations:

$$\nabla.\mathbf{E} = \rho/\epsilon_0$$

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$$\nabla.\mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \dot{\mathbf{E}}.$$

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$$\nabla.\mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \mu_0\mathbf{J} + \mu_0\epsilon_0\dot{\mathbf{E}}.$$

This means that:

- The electric field diverges in regions where there is positive charge, and converges in regions where there is negative charge.

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This means that:

- The electric field diverges in regions where there is positive charge, and converges in regions where there is negative charge.
- The magnetic field never diverges or converges.

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- The magnetic field $\mathbf{B}$, which is another vector field.
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This means that:

- The electric field diverges in regions where there is positive charge, and converges in regions where there is negative charge.
- The magnetic field never diverges or converges.
- Changing magnetic fields cause the electric field to curl.

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The quantities $\mathbf{E}$, $\mathbf{B}$, $\mathbf{J}$ and ρ may also depend on time; we write $\dot{\mathbf{E}}$ for $\partial\mathbf{E}/\partial t$ and so on. The various fields are related by the following equations:

$$\nabla.\mathbf{E} = \rho/\epsilon_0 \qquad \nabla \times \mathbf{E} = -\dot{\mathbf{B}}$$

$$\nabla.\mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \mu_0\mathbf{J} + \mu_0\epsilon_0\dot{\mathbf{E}}.$$

This means that:

- The electric field diverges in regions where there is positive charge, and converges in regions where there is negative charge.
- The magnetic field never diverges or converges.
- Changing magnetic fields cause the electric field to curl.
- Currents cause the magnetic field to curl.

Maxwell's equations

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The quantities **E**, **B**, **J** and ρ may also depend on time; we write $\dot{\mathbf{E}}$ for $\partial \mathbf{E}/\partial t$ and so on. The various fields are related by the following equations:

$$\nabla.\mathbf{E} = \rho/\epsilon_0 \qquad \nabla \times \mathbf{E} = -\dot{\mathbf{B}}$$
$$\nabla.\mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \dot{\mathbf{E}}.$$

This means that:

- The electric field diverges in regions where there is positive charge, and converges in regions where there is negative charge.
- The magnetic field never diverges or converges.
- Changing magnetic fields cause the electric field to curl.
- Currents cause the magnetic field to curl. Changing electric fields also cause the magnetic field to curl, but the effect is usually much weaker, because ϵ_0 is small.

Plane wave solution to Maxwell's equations

One class of solutions to Maxwell's equations is as follows.

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Plane wave solution to Maxwell's equations

One class of solutions to Maxwell's equations is as follows. Put $c = 1/\sqrt{\mu_0 \epsilon_0} \simeq 3 \times 10^8 ms^{-1}$ (which turns out to be the speed of light), and let α be any constant.

Plane wave solution to Maxwell's equations

One class of solutions to Maxwell's equations is as follows. Put $c = 1/\sqrt{\mu_0\epsilon_0} \simeq 3 \times 10^8 ms^{-1}$ (which turns out to be the speed of light), and let α be any constant. We can take $\mathbf{J} = 0$ and $\rho = 0$ and

$$\mathbf{E} = (0, \sin(\alpha(x - ct)), 0) \qquad \mathbf{B} = (0, 0, \sin(\alpha(x - ct))/c).$$

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We find that

$$\nabla.\mathbf{E} = \frac{\partial}{\partial y} \sin(\alpha(x - ct))$$

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$$\mathbf{E} = (0, \sin(\alpha(x - ct)), 0) \qquad \mathbf{B} = (0, 0, \sin(\alpha(x - ct))/c).$$

We find that

$$\nabla.\mathbf{E} = \frac{\partial}{\partial y} \sin(\alpha(x - ct)) = 0$$

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$$\mathbf{E} = (0, \sin(\alpha(x - ct)), 0) \qquad \mathbf{B} = (0, 0, \sin(\alpha(x - ct))/c).$$

We find that

$$\nabla.\mathbf{E} = \frac{\partial}{\partial y}\sin(\alpha(x - ct)) = 0 = \rho/\epsilon_0 \qquad \dot{\mathbf{E}} = (0, -\alpha c\cos(\alpha(x - ct)), 0)$$

$$\nabla.\mathbf{B} = \frac{\partial}{\partial z}\sin(\alpha(x - ct))/c = 0 \qquad \dot{\mathbf{B}} = (0, 0, -\alpha\cos(\alpha(x - ct))$$

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$$\nabla.\mathbf{B} = \frac{\partial}{\partial z}\sin(\alpha(x - ct))/c = 0 \qquad \dot{\mathbf{B}} = (0, 0, -\alpha\cos(\alpha(x - ct))$$

$$\nabla \times \mathbf{E} = \det\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & \sin(\alpha(x - ct)) & 0 \end{bmatrix}$$

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$$\nabla.\mathbf{B} = \frac{\partial}{\partial z}\sin(\alpha(x - ct))/c = 0 \qquad \dot{\mathbf{B}} = (0, 0, -\alpha\cos(\alpha(x - ct))$$

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$$\nabla \times \mathbf{E} = \det\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & \sin(\alpha(x - ct)) & 0 \end{bmatrix} = (0, 0, \alpha\cos(\alpha(x - ct))) = -\dot{\mathbf{B}}$$

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$$\nabla \times \mathbf{E} = \det\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & \sin(\alpha(x - ct)) & 0 \end{bmatrix} = (0, 0, \alpha\cos(\alpha(x - ct))) = -\dot{\mathbf{B}}$$

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Plane wave solution to Maxwell's equations

One class of solutions to Maxwell's equations is as follows. Put $c = 1/\sqrt{\mu_0 \epsilon_0} \simeq 3 \times 10^8 ms^{-1}$ (which turns out to be the speed of light), and let α be any constant. We can take $\mathbf{J} = 0$ and $\rho = 0$ and

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This shows that we do indeed have a solution to the equations.

Plane wave solution to Maxwell's equations

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This shows that we do indeed have a solution to the equations. It represents an electromagnetic wave of wavelength $1/\alpha$ moving at speed c in the x-direction.

Stationary charged particle

Another solution to Maxwell's equations has $\mathbf{E} = (-xr^{-3}, -yr^{-3}, -zr^{-3})$ with all other fields ($\mathbf{B}$, $\mathbf{J}$ and ρ) being zero.

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$$\nabla.\mathbf{E} = (-xr^{-3})_x + (-yr^{-3})_y + (-zr^{-3})_z$$

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$$
\begin{aligned}
\nabla.\mathbf{E} &= (-xr^{-3})_x + (-yr^{-3})_y + (-zr^{-3})_z \\
&= 3x^2r^{-5} - r^{-3} + 3y^2r^{-5} - r^{-3} + 3z^2r^{-5} - r^{-3} \\
&= 3(x^2 + y^2 + z^2)r^{-5} - 3r^{-3}
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\end{aligned}$$

$$\nabla \times \mathbf{E} = \left((-zr^{-3})_y - (-yr^{-3})_z, (-xr^{-3})_z - (-zr^{-3})_x, (-yr^{-3})_x - (-xr^{-3})_y\right)$$

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\end{aligned}
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\nabla \times \mathbf{E} &= \left((-zr^{-3})_y - (-yr^{-3})_z, (-xr^{-3})_z - (-zr^{-3})_x, (-yr^{-3})_x - (-xr^{-3})_y\right) \\
&= \left(3yzr^{-3} - 3yzr^{-3}, 3xzr^{-3} - 3xzr^{-3}, 3xyr^{-3} - 3xyr^{-3}\right)
\end{aligned}
$$

Stationary charged particle

Another solution to Maxwell's equations has $\mathbf{E} = (-xr^{-3}, -yr^{-3}, -zr^{-3})$ with all other fields ($\mathbf{B}$, $\mathbf{J}$ and ρ) being zero. It is clear that $\dot{\mathbf{E}} = 0$ and $\dot{\mathbf{B}} = 0$, so the only equations that we need to check are that $\nabla.\mathbf{E} = 0$ and $\nabla \times \mathbf{E} = 0$. For this we recall that $r_x = x/r$, so $(r^{-3})_x = -3r^{-4}r_x = -3xr^{-5}$. In the same way, we have $(r^{-3})_y = -3yr^{-5}$ and $(r^{-3})_z = -3zr^{-5}$. Using this we find that

$$
\begin{aligned}
(-xr^{-3})_x &= 3x^2r^{-5} - r^{-3} & (-xr^{-3})_y &= 3xyr^{-5} & (-xr^{-3})_z &= 3xzr^{-5} \\
(-yr^{-3})_x &= 3xyr^{-5} & (-yr^{-3})_y &= 3y^2r^{-5} - r^{-3} & (-yr^{-3})_z &= 3yzr^{-5} \\
(-zr^{-3})_x &= 3xzr^{-5} & (-zr^{-3})_y &= 3yzr^{-5} & (-zr^{-3})_z &= 3z^2r^{-5} - r^{-3}.
\end{aligned}
$$

$$
\begin{aligned}
\nabla.\mathbf{E} &= (-xr^{-3})_x + (-yr^{-3})_y + (-zr^{-3})_z \\
&= 3x^2r^{-5} - r^{-3} + 3y^2r^{-5} - r^{-3} + 3z^2r^{-5} - r^{-3} \\
&= 3(x^2 + y^2 + z^2)r^{-5} - 3r^{-3} = 3r^2r^{-5} - 3r^{-3} = 0.
\end{aligned}
$$

$$
\begin{aligned}
\nabla \times \mathbf{E} &= \left((-zr^{-3})_y - (-yr^{-3})_z, (-xr^{-3})_z - (-zr^{-3})_x, (-yr^{-3})_x - (-xr^{-3})_y\right) \\
&= \left(3yzr^{-3} - 3yzr^{-3}, 3xzr^{-3} - 3xzr^{-3}, 3xyr^{-3} - 3xyr^{-3}\right) = (0,0,0).
\end{aligned}
$$

Another solution to Maxwell's equations has $\mathbf{E} = (-xr^{-3}, -yr^{-3}, -zr^{-3})$ with all other fields ($\mathbf{B}$, $\mathbf{J}$ and ρ) being zero. It is clear that $\dot{\mathbf{E}} = 0$ and $\dot{\mathbf{B}} = 0$, so the only equations that we need to check are that $\nabla.\mathbf{E} = 0$ and $\nabla \times \mathbf{E} = 0$. For this we recall that $r_x = x/r$, so $(r^{-3})_x = -3r^{-4}r_x = -3xr^{-5}$. In the same way, we have $(r^{-3})_y = -3yr^{-5}$ and $(r^{-3})_z = -3zr^{-5}$. Using this we find that

$$\begin{array}{lll} (-xr^{-3})_x = 3x^2r^{-5} - r^{-3} & (-xr^{-3})_y = 3xyr^{-5} & (-xr^{-3})_z = 3xzr^{-5} \\ (-yr^{-3})_x = 3xyr^{-5} & (-yr^{-3})_y = 3y^2r^{-5} - r^{-3} & (-yr^{-3})_z = 3yzr^{-5} \\ (-zr^{-3})_x = 3xzr^{-5} & (-zr^{-3})_y = 3yzr^{-5} & (-zr^{-3})_z = 3z^2r^{-5} - r^{-3}. \end{array}$$

$$\begin{aligned} \nabla.\mathbf{E} &= (-xr^{-3})_x + (-yr^{-3})_y + (-zr^{-3})_z \\ &= 3x^2r^{-5} - r^{-3} + 3y^2r^{-5} - r^{-3} + 3z^2r^{-5} - r^{-3} \\ &= 3(x^2 + y^2 + z^2)r^{-5} - 3r^{-3} = 3r^2r^{-5} - 3r^{-3} = 0. \end{aligned}$$

$$\begin{aligned} \nabla \times \mathbf{E} &= \left((-zr^{-3})_y - (-yr^{-3})_z, (-xr^{-3})_z - (-zr^{-3})_x, (-yr^{-3})_x - (-xr^{-3})_y\right) \\ &= \left(3yzr^{-3} - 3yzr^{-3}, 3xzr^{-3} - 3xzr^{-3}, 3xyr^{-3} - 3xyr^{-3}\right) = (0, 0, 0). \end{aligned}$$

This shows that we have a solution to the equations, as claimed.

Stationary charged particle

Another solution to Maxwell's equations has $\mathbf{E} = (-xr^{-3}, -yr^{-3}, -zr^{-3})$ with all other fields ($\mathbf{B}$, $\mathbf{J}$ and ρ) being zero. It is clear that $\dot{\mathbf{E}} = 0$ and $\dot{\mathbf{B}} = 0$, so the only equations that we need to check are that $\nabla.\mathbf{E} = 0$ and $\nabla \times \mathbf{E} = 0$. For this we recall that $r_x = x/r$, so $(r^{-3})_x = -3r^{-4}r_x = -3xr^{-5}$. In the same way, we have $(r^{-3})_y = -3yr^{-5}$ and $(r^{-3})_z = -3zr^{-5}$. Using this we find that

$$
\begin{aligned}
(-xr^{-3})_x &= 3x^2r^{-5} - r^{-3} & (-xr^{-3})_y &= 3xyr^{-5} & (-xr^{-3})_z &= 3xzr^{-5} \\
(-yr^{-3})_x &= 3xyr^{-5} & (-yr^{-3})_y &= 3y^2r^{-5} - r^{-3} & (-yr^{-3})_z &= 3yzr^{-5} \\
(-zr^{-3})_x &= 3xzr^{-5} & (-zr^{-3})_y &= 3yzr^{-5} & (-zr^{-3})_z &= 3z^2r^{-5} - r^{-3}.
\end{aligned}
$$

$$
\begin{aligned}
\nabla.\mathbf{E} &= (-xr^{-3})_x + (-yr^{-3})_y + (-zr^{-3})_z \\
&= 3x^2r^{-5} - r^{-3} + 3y^2r^{-5} - r^{-3} + 3z^2r^{-5} - r^{-3} \\
&= 3(x^2 + y^2 + z^2)r^{-5} - 3r^{-3} = 3r^2r^{-5} - 3r^{-3} = 0.
\end{aligned}
$$

$$
\begin{aligned}
\nabla \times \mathbf{E} &= \left((-zr^{-3})_y - (-yr^{-3})_z, (-xr^{-3})_z - (-zr^{-3})_x, (-yr^{-3})_x - (-xr^{-3})_y\right) \\
&= \left(3yzr^{-3} - 3yzr^{-3}, 3xzr^{-3} - 3xzr^{-3}, 3xyr^{-3} - 3xyr^{-3}\right) = (0, 0, 0).
\end{aligned}
$$

This shows that we have a solution to the equations, as claimed. This one represents the electric field of a single stationary particle at the origin, with no magnetic field.