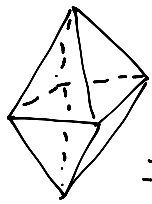
MAS435 Algebraic Topology.

Semester 2 · Weekly Solutions 1

(a)



Vertices: $(\pm 1,0,0), (0,\pm 1,0), (0,0,\pm 1)$
$\pm e_1$ $\pm e_2$ $\pm e_3$

Facets: $\{\varepsilon_1 e_1, \varepsilon_2 e_2, \varepsilon_3 e_3\}$ for $\varepsilon_i \in \{-1,1\}$

In $\mathbb{R}^n$ vertices $\pm e_i$ ($2n$ of them)

$(n-1)$ dim'l faces: $\{\varepsilon_1 e_1, \varepsilon_2 e_2, \dots, \varepsilon_n e_n\}$ for $\varepsilon_i \in \{-1,1\}$ (2^n of them)

(b) Suppose $v_1 - v_0, \dots, v_k - v_0$ are linearly independent

Note
$$\mu_0(v_0 - v_1) + \mu_2(v_2 - v_1) + \cdots + \mu_k(v_k - v_1)$$
$$=$$
$$\lambda_1(v_1 - v_0) + \lambda_2(v_2 - v_0) + \cdots + \lambda_k(v_k - v_0)$$

where
$$\lambda_1 = -\mu_0 - \mu_2 - \cdots - \mu_k \qquad \mu_0 = -\lambda_1 - \lambda_2 - \cdots - \lambda_k$$
$$\lambda_2 = \mu_2 \qquad \mu_2 = \lambda_2$$
$$\vdots \qquad \vdots$$
$$\lambda_k = \mu_k \qquad \mu_k = \lambda_k$$

The condition that $\lambda_i = 0$ for all i is therefore equivalent to the condition that $\mu_i = 0$ for all i

(c) 2: $\mathbb{Z}/2$
4: $\mathbb{Z}/4$ or $\mathbb{Z}/2 \times \mathbb{Z}/2$
8: $\mathbb{Z}/8$ or $\mathbb{Z}/4 \times \mathbb{Z}/2$ or $\mathbb{Z}/2 \times \mathbb{Z}/2 \times \mathbb{Z}/2$

6: $\mathbb{Z}/6$ (all abelian groups of this order are cyclic)
10: $\mathbb{Z}/10$ —— " ——
15: $\mathbb{Z}/15$ —— " ——