

MAS435 Algebraic Topology
Semester 2 Weekly Submis 4

$0 \rightarrow 0 \rightarrow \mathbb{Z} \xrightarrow{6} \mathbb{Z} \rightarrow 0$: Only one nonzero map so $d^2=0$

$H_x: 0 \quad 0 \quad \mathbb{Z}/6$

$0 \rightarrow \mathbb{Z}/4 \xrightarrow{2} \mathbb{Z}/4 \xrightarrow{3} \mathbb{Z}/6 \rightarrow 0$: The one pair of maps has composite 6 in $\mathbb{Z}/6$, so $d^2=0$

$H_x: \mathbb{Z}/2 \quad 0 \quad \mathbb{Z}/3$

$\mathbb{Z} \xrightarrow{2} \mathbb{Z} \xrightarrow{2} \mathbb{Z} \xrightarrow{2} \mathbb{Z} \xrightarrow{2} \mathbb{Z} \xrightarrow{2} \mathbb{Z}$: The composites d^2 are multiplication by 4 & hence nonzero. Not a chain complex

$\mathbb{Z} \xrightarrow{2} \mathbb{Z}/4 \xrightarrow{2} \mathbb{Z}/4 \xrightarrow{2} \mathbb{Z}/4 \xrightarrow{2} \mathbb{Z}/4 \xrightarrow{2} \mathbb{Z}/4$ The composites d^2 are multiplication by 4 & hence zero

$H_x: 0 \quad 0 \quad 0 \quad 0 \quad 0$ (an exact sequence).

$0 \rightarrow \mathbb{Z} \xrightarrow{0} \mathbb{Z} \xrightarrow{2} \mathbb{Z} \xrightarrow{0} \mathbb{Z} \xrightarrow{2} \mathbb{Z} \xrightarrow{0}$: All 2-fold composites involve a zero, so $d^2=0$

$H_x: \mathbb{Z} \quad 0 \quad \mathbb{Z}/2 \quad 0 \quad \mathbb{Z}/2$ (Aside: $H^*(\mathbb{R}P^\infty; \mathbb{Z}) = \mathbb{Z}[x]/(2x)$)

$0 \rightarrow \mathbb{Z} \xrightarrow{2} \mathbb{Z} \xrightarrow{0} \mathbb{Z} \xrightarrow{2} \mathbb{Z} \xrightarrow{0} \mathbb{Z} \xrightarrow{2}$: All 2-fold composites include a zero, so $d^2=0$

$H_x: 0 \quad \mathbb{Z} \quad 0 \quad \mathbb{Z} \quad 0$ (Aside: $H^*(\mathbb{C}_2; \mathbb{Z})$)