

# MAS435 Quiz

Name:

1. Give the definition of a continuous map  $f: X \rightarrow Y$  between topological spaces  $X$  and  $Y$ .

2. Show by drawing a picture how each of the following spaces can be obtained by identifying sides of a square.

(a) The torus

(b) The real projective plane

(c) The Klein bottle

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1. If  $\gamma_1: [0,1] \rightarrow X$  is a path from  $x$  to  $y$  and  $\gamma_2: [0,1] \rightarrow X$  is a path from  $y$  to  $z$  in a space  $X$ , give a formula for the concatenation  $\gamma_1 \cdot \gamma_2: [0,1] \rightarrow X$ . (2 points)

2. If  $\vec{u}, \vec{v} \in \mathbb{R}^2$  are any vectors, give a formula for a path  $\gamma: [0,1] \rightarrow \mathbb{R}^2$  from  $\vec{u}$  to  $\vec{v}$ . (2 points)

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1. Define what it means for two maps  $f, g: X \rightarrow Y$  to be homotopic. (2 points)

2. Let  $X$  be a space and  $Y \subseteq X$  a subspace. Define what it means for a map  $r: X \rightarrow Y$  to be a deformation retraction. (2 points)

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Define the notion of a category. (4 points)

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1. If  $f: X \rightarrow Y$  is continuous, define the induced map  $f_*: \pi_1(X, x_0) \rightarrow \pi_1(Y, f(x_0))$ : (1 point)
2. State the path lifting lemma for the covering map  $p: \mathbb{R} \rightarrow S^1$ ,  $t \mapsto e^{2\pi i t}$ . (3 points)

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State the homotopy lifting lemma for a general covering map  $p: \tilde{X} \rightarrow X$ . (4 points)

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Write down the fundamental group of each of the following spaces. (Each is worth 1 point; you do not need to justify your answers.)

1. The sphere  $S^2$

2. The bouquet of  $n$  circles  $\bigvee_n S^1$

3. The torus  $T^2$

4. Real projective space  $\mathbb{R}P^n$  for  $n \geq 2$