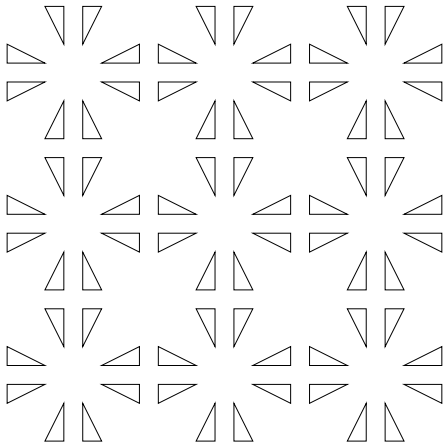
Groups and Symmetry Problem Set 2

Q1: Note that $R_\pi(x) = -x$. Give expressions for $R_{a,\pi}(x)$, $R_{a,\pi}^{-1}(x)$ and $R_{a,\pi}R_{b,\pi}R_{a,\pi}^{-1}R_{b,\pi}^{-1}(x)$. Check that your last answer is consistent with the formula in the notes for $R_{a,\theta}R_{b,\phi}R_{a,\theta}^{-1}R_{b,\phi}^{-1}$.

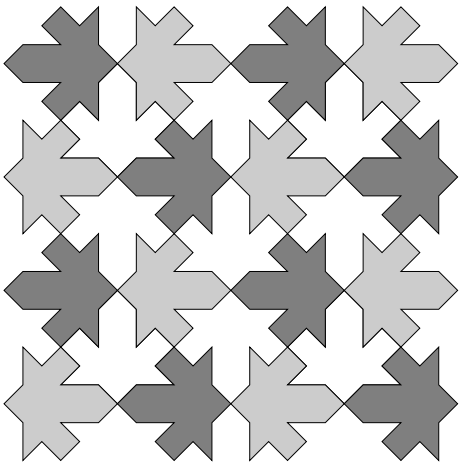
Q2: Three infinite wall-paper patterns are represented below by a small segment of the pattern.

(a) Find the isometry group of pattern (a).
(b) Find the isometry group of pattern (b).
(c) Find the isometry group of pattern (c) and show that it is generated by a reflection and a rotation.

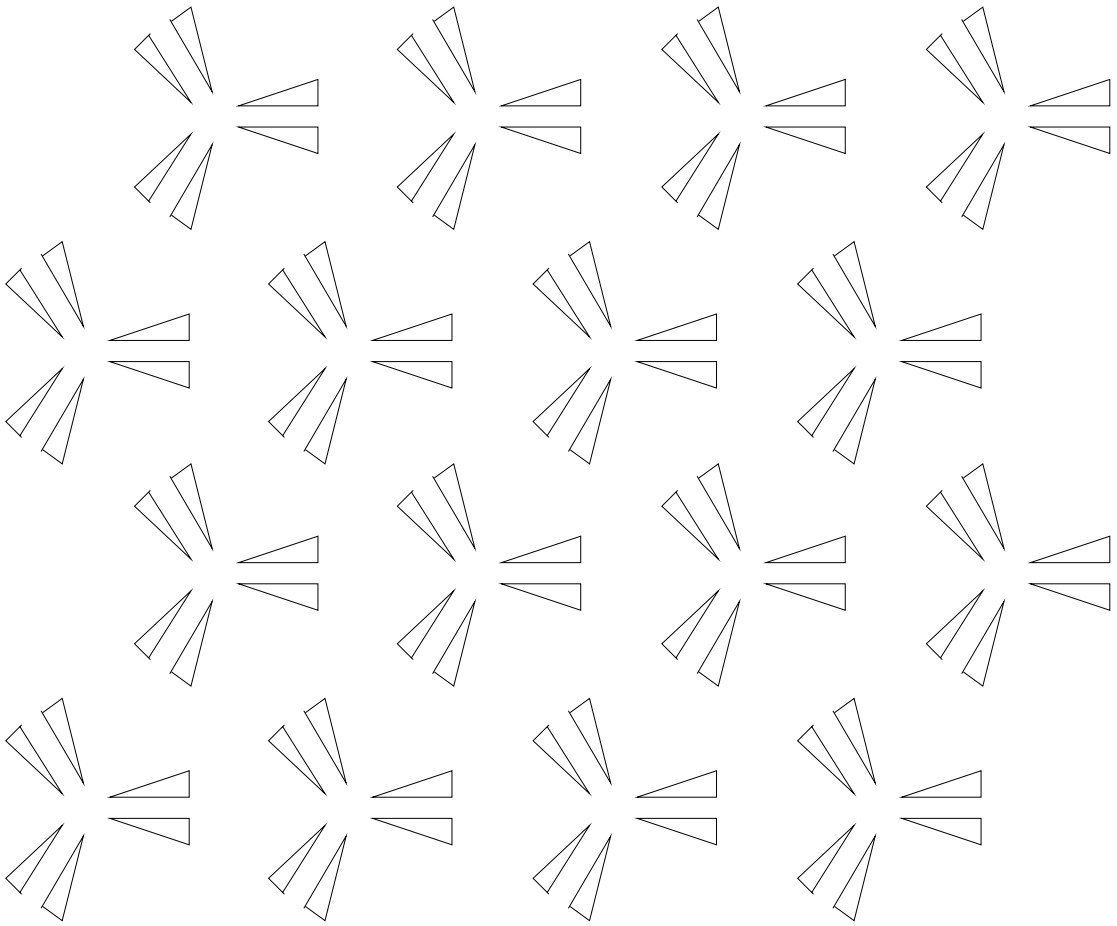
Pattern (a):



Pattern (b):



Pattern (c):



Q3: Let T be a triangle with angles $\pi/2$, $\pi/6$ and $\pi/3$. Let S_l, S_m and S_n be the reflections in the three sides of T. Find a pattern X with isometry group $\mathrm{Isom}(X)$ generated by S_l, S_m and S_n.