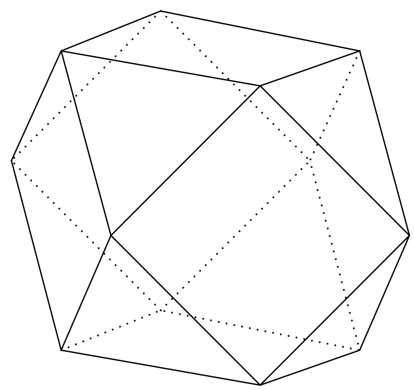
Groups and Symmetry Problem Set 5

Please hand in questions 1 and 4 on Tuesday 25th November.

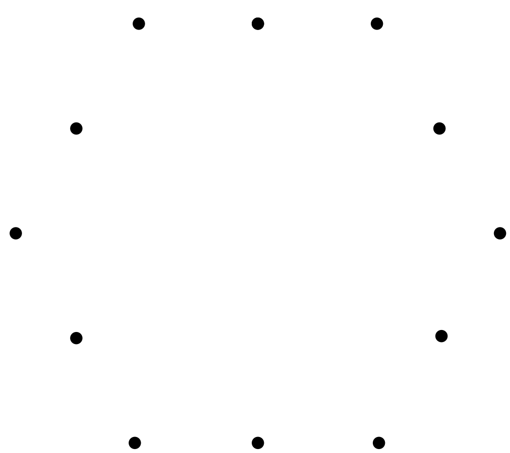
Q1: The following diagram shows a cuboctahedron X in $\mathbb{R}^3$, centred at the origin. Its faces are squares and equilateral triangles.



Which of the standard finite subgroups of SO_3 is isomorphic to $\mathrm{Dir}(X)$? What can you deduce about $\mathrm{Symm}(X)$?

Q2: The group Dir(Cube) of rotational symmetries of the cube acts on the surface of the cube. Find the sizes of the orbits of points on the surface and describe geometrically which points have which orbit sizes.

Q3: Let X be the following subset of $\mathbb{R}^2$ (with centre at the origin). The group D_3 acts on X. Find the orbits of this action, find the fixed points of all the elements of D_3, and verify the orbit counting theorem.



Q4: Let G be a group of order 77, and let X be a set of order 96 on which G acts. Suppose there are precisely four orbits. By investigating the possible sizes of the orbits, show that there is exactly one point $x \in X$ such that $gx = x$ for all $g \in G$.

Q5: By considering cycle types show that A_5 has no elements of order 15. What does the classification of finite subgroups of SO_3 tell us about subgroups of A_5 of order 30? Show that there are no such subgroups.