## Simplicial complexes

A *geometric simplicial complex* is a space formed from a finite collection of triangles, tetrahedra and their higher-dimensional analogues, which fit together nicely. An *abstract simplicial complex* consists of a finite set $V$ of *vertices* together with a set $K$ of nonempty subsets $\sigma \subseteq V$ of *simplices*, such that every nonempty subset of a simplex is another simplex. One can build a geometric complex from an abstract one (this is called *geometric realisation*), so there is a sense in which the two concepts are equivalent, although some thought is needed to make this precise. For many interesting metric spaces $X$ (including all the examples in the algebraic topology course) one can find an abstract simplicial complex whose geometric realisation is homotopy equivalent (often even homeomorphic) to $X$. As an abstract simplicial complex is a finite, combinatorial object, this can be a very convenient representation. [1]

Here are some possible ingredients of a project. (One would not expect to cover all of them.)

- The proof (involving some nice combinatorics) that the product of two simplicial complexes is a simplicial complex.
- Abstract and geometric versions of subdivision (cutting simplices into smaller ones) and proof that they are compatible.
- Abstract simplicial maps give continuous maps of geometric realisations. Geometric realisations of contiguous maps are homotopic.
- The set of chains in a partially ordered set give an abstract simplicial complex. Examples, possibly including Quillen's work on partially ordered sets of $p$-subgroups in a finite group.
- The fundamental group and the homology groups of a simplicial complex. Contiguous maps have the same effect in homology.
- Connections with topics in the Knots and Surfaces course.

There are some pictures to illustrate these ideas at the following URL:
`http://www.shef.ac.uk/nps/projects/simplicial`.

### References

[1] C. R. F. Maunder, *Algebraic topology*, Dover Publications Inc., Mineola, NY, 1996. Reprint of the 1980 edition.MR 97c:55001

[2] Rudolf Fritsch and Renzo A. Piccinini, *Cellular structures in topology*, Cambridge Studies in Advanced Mathematics, vol. 19, Cambridge University Press, Cambridge, 1990.MR 92d:55001

[3] James R. Munkres, *Elements of algebraic topology*, Addison-Wesley Publishing Company, Menlo Park, CA, 1984.MR 85m:55001

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[1] As well as these connections with pure mathematics via topology, there are numerous uses for simplices in engineering (the "finite element method") and computer graphics. However, the project supervisor is no expert on these ideas.