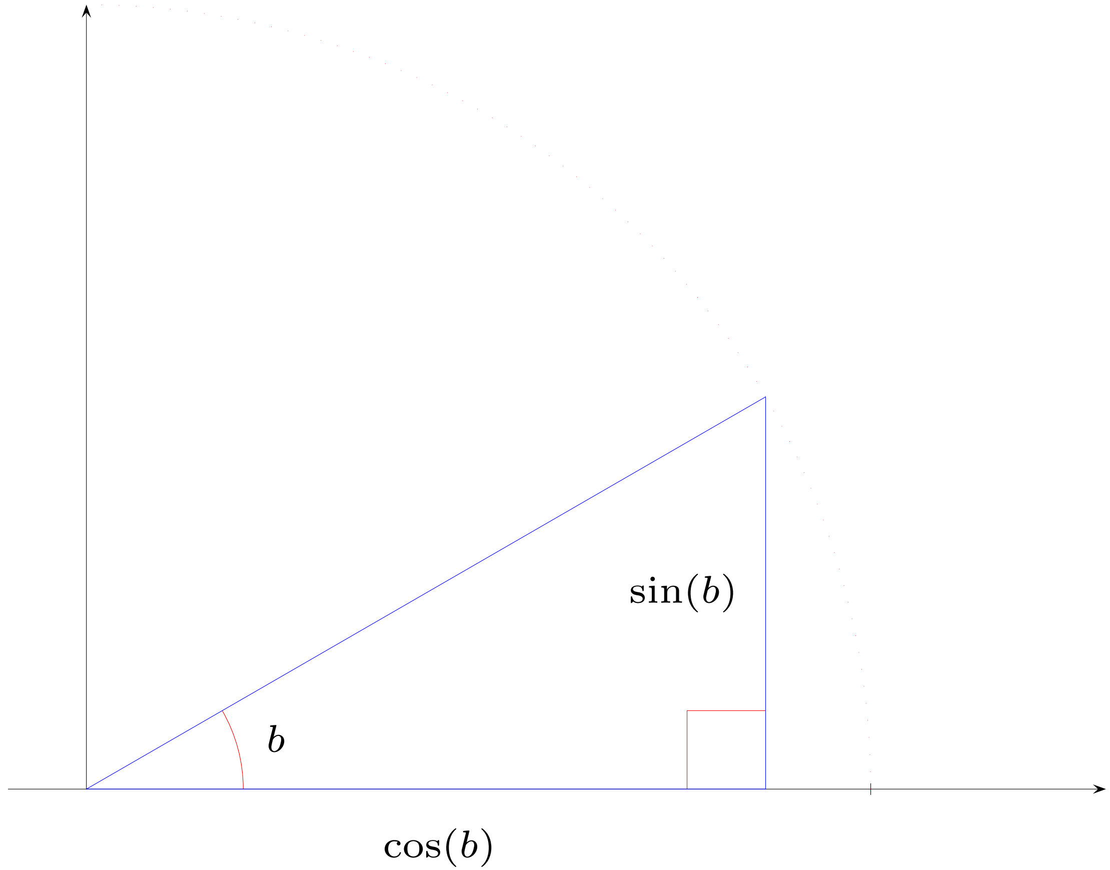
The addition formula

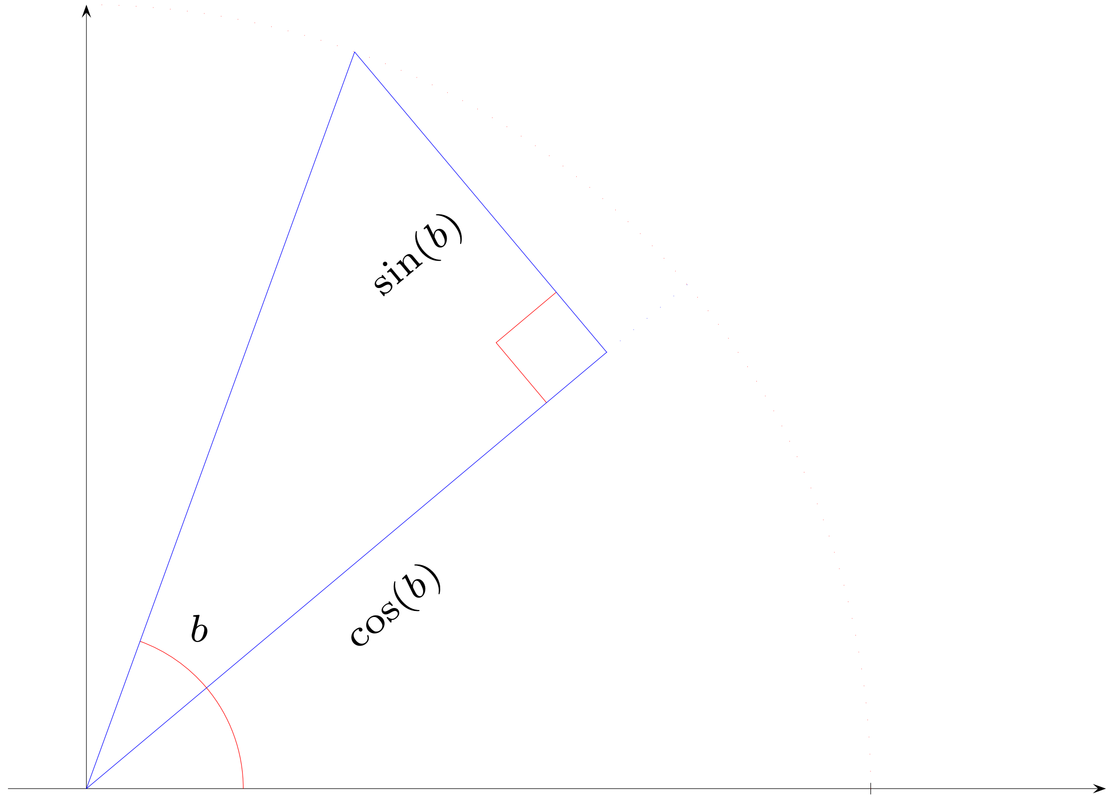
The addition formula

- $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$

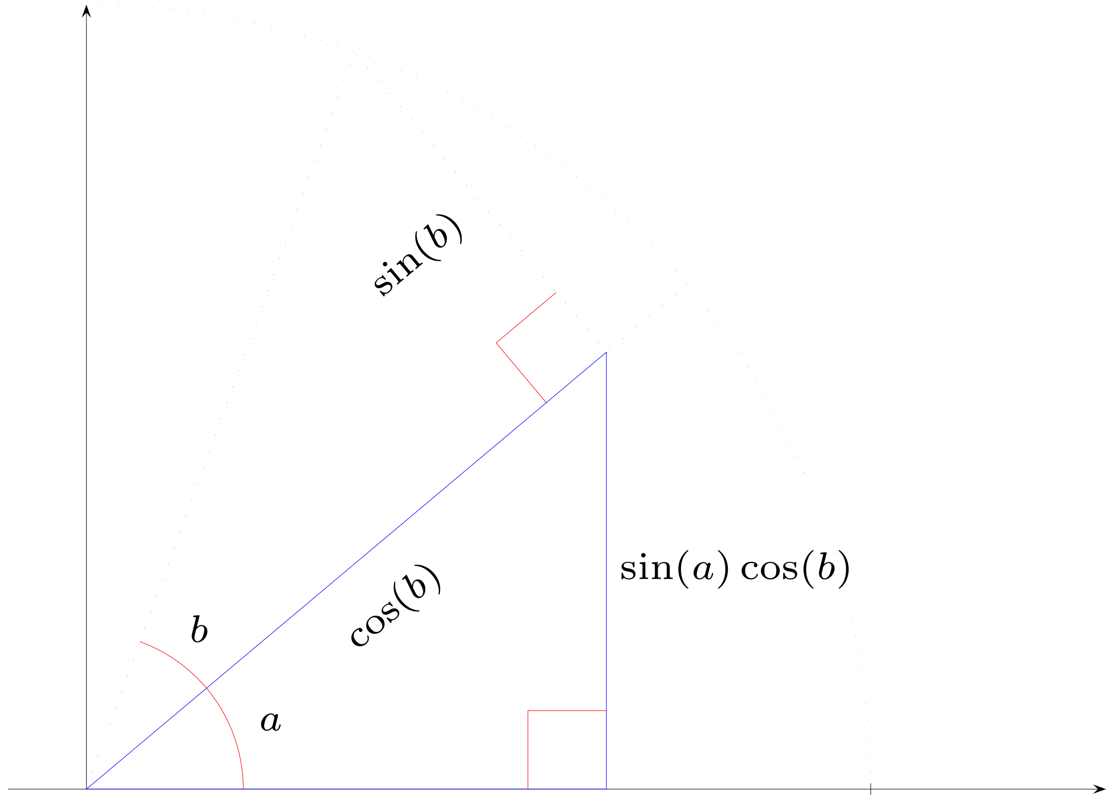
The addition formula

- $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$
- 

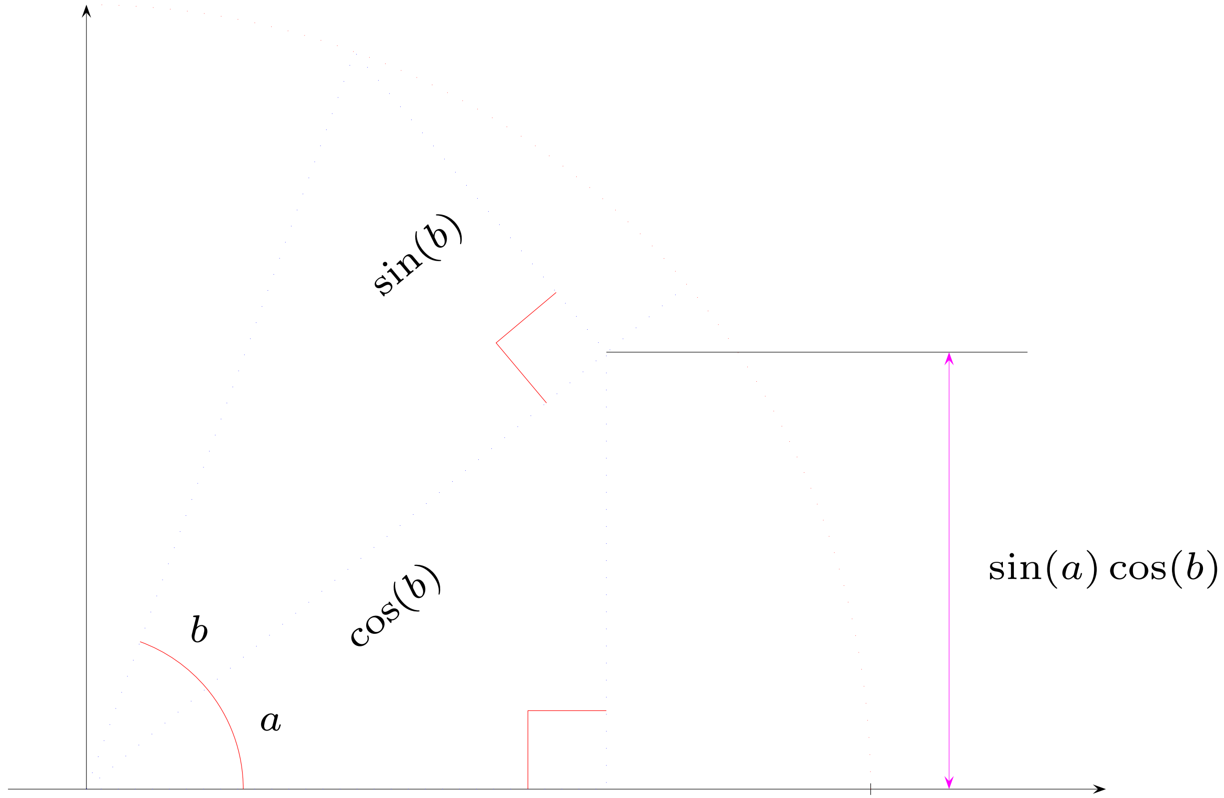
The addition formula

- $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$
- 

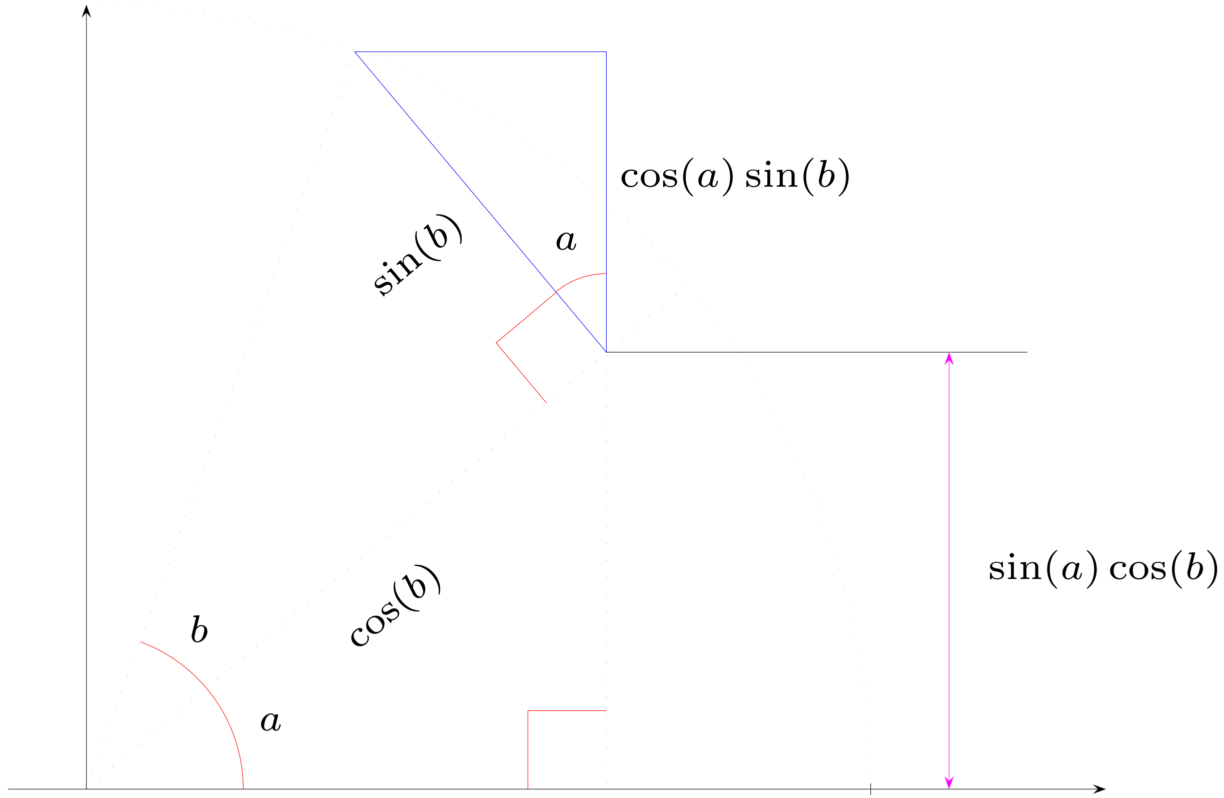
The addition formula

- $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$
- 

The addition formula

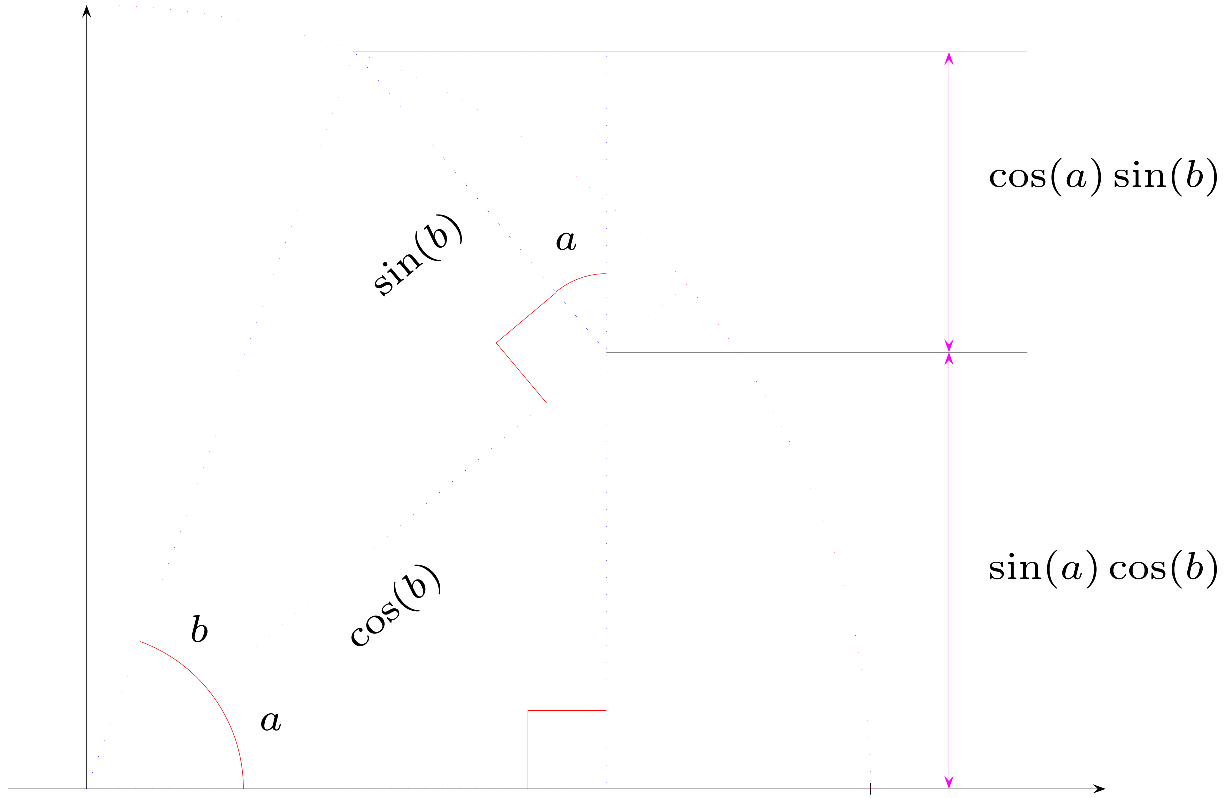
- $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$
- 

The addition formula

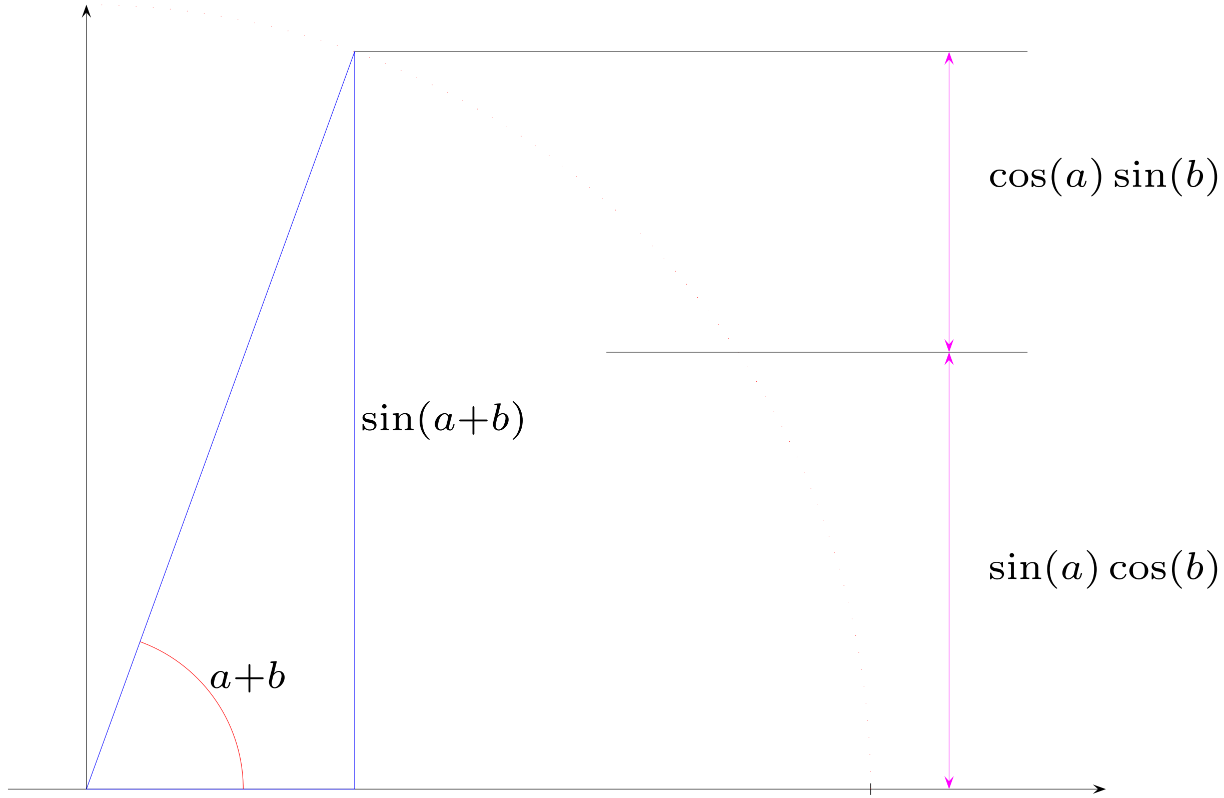
- $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$
- 

The addition formula

- $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$
-

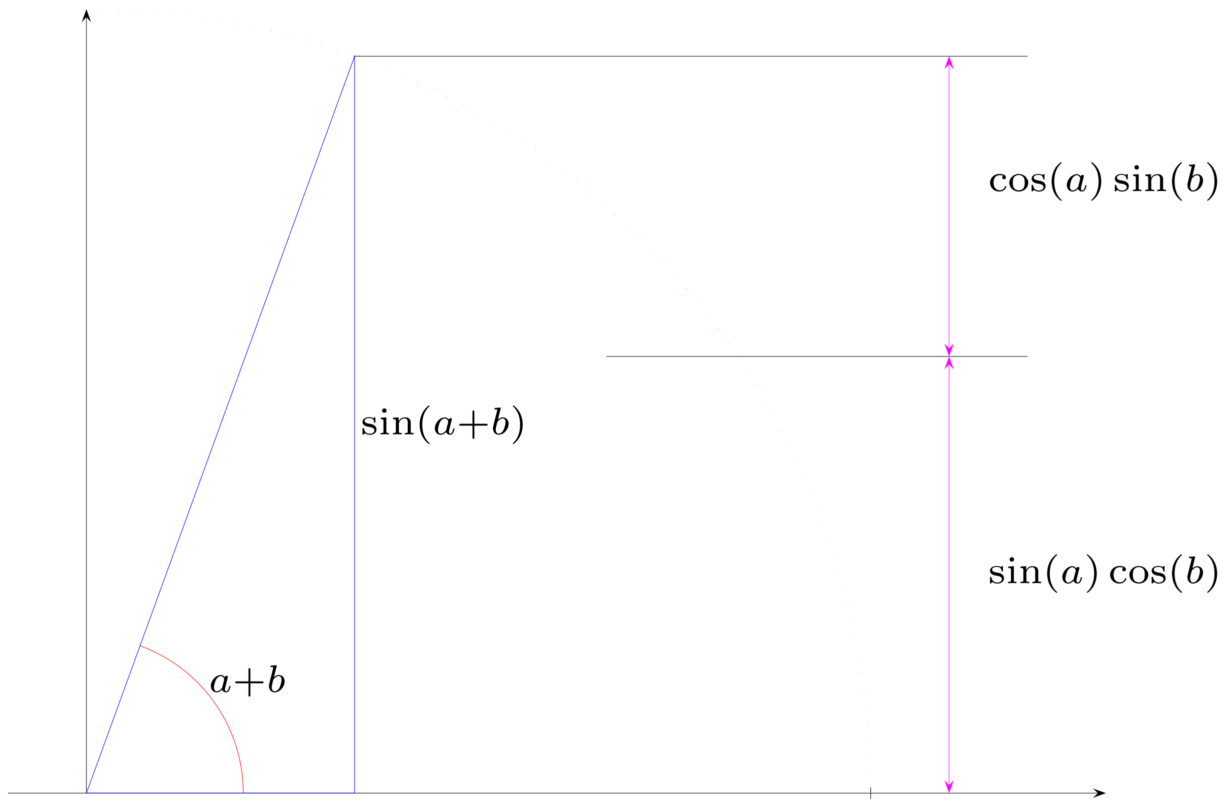


The addition formula

- $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$
- 

The addition formula

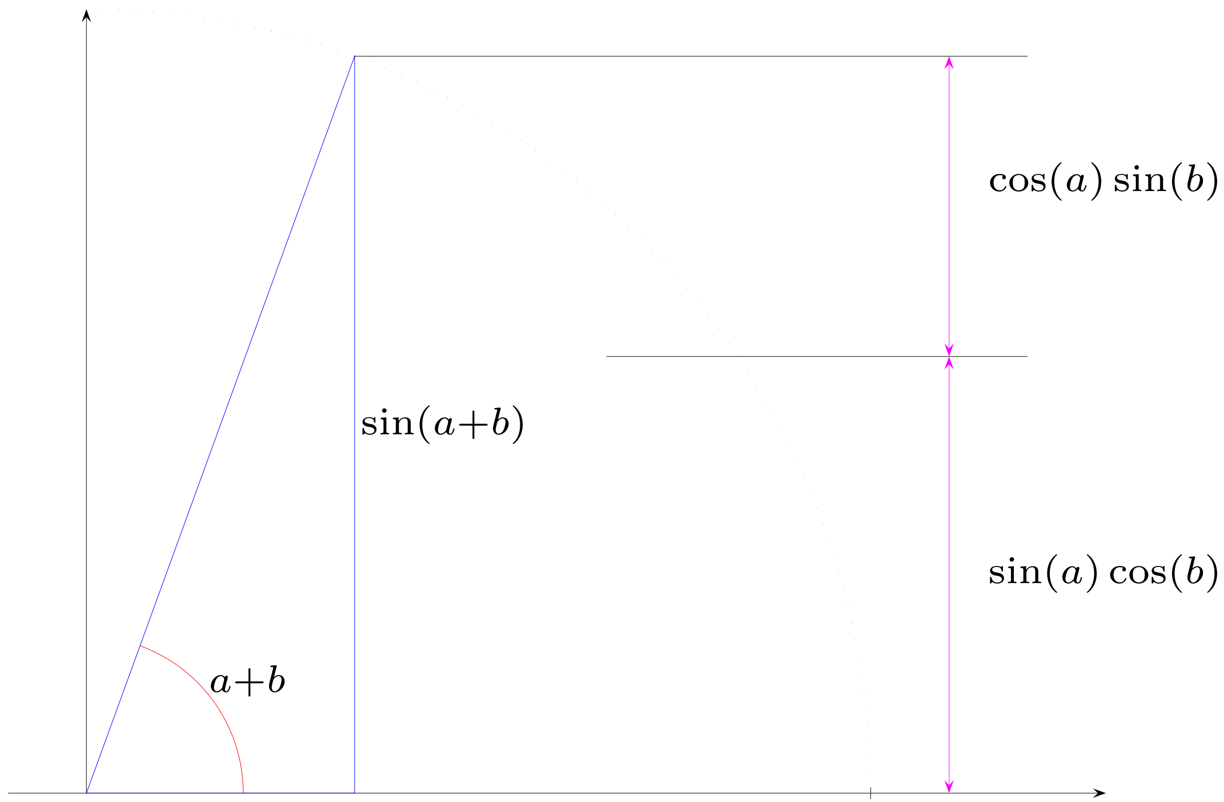
- $$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

- 

- $$\sin(a)\cos(b) + \cos(a)\sin(b) = \frac{e^{ia} - e^{-ia}}{2i}\frac{e^{ib} + e^{-ib}}{2} + \frac{e^{ia} + e^{-ia}}{2}\frac{e^{ib} - e^{-ib}}{2i}$$

The addition formula

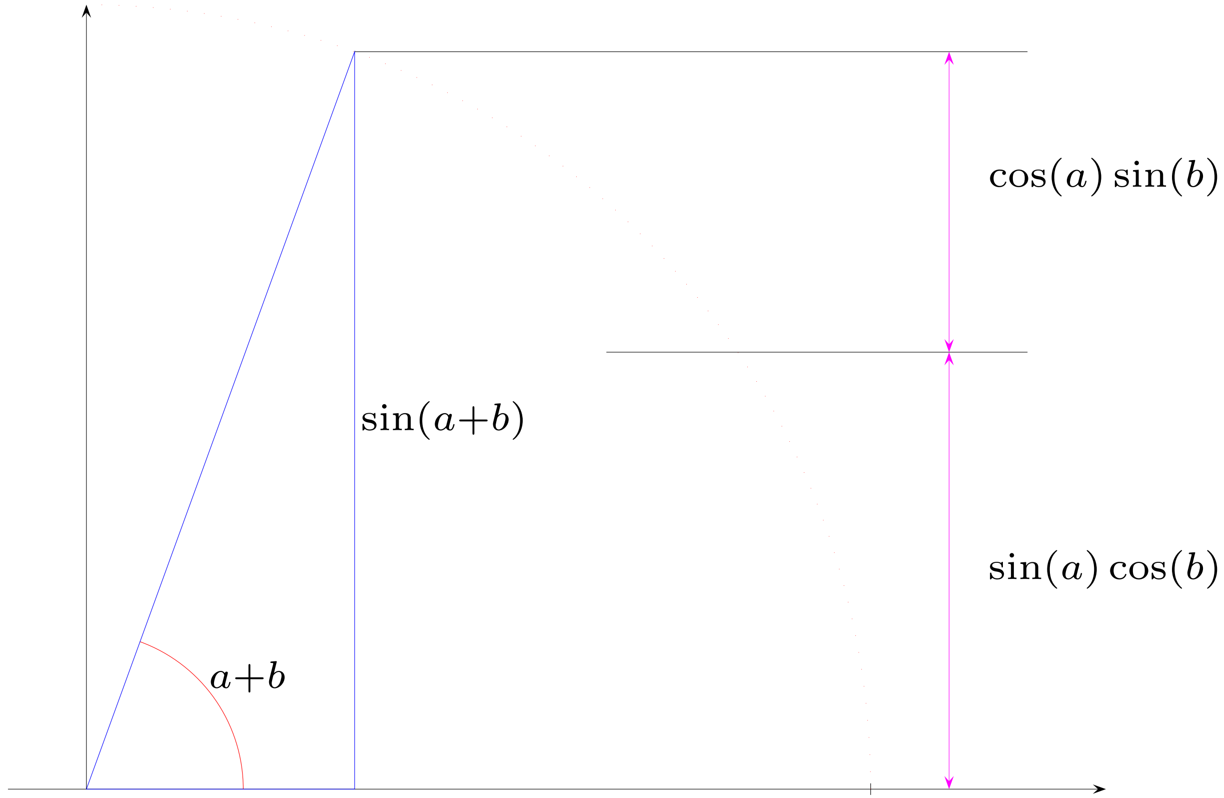
- $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$

- 

- $$\sin(a)\cos(b) + \cos(a)\sin(b) = \frac{e^{ia} - e^{-ia}}{2i}\frac{e^{ib} + e^{-ib}}{2} + \frac{e^{ia} + e^{-ia}}{2}\frac{e^{ib} - e^{-ib}}{2i}$$
$$= \frac{e^{i(a+b)} - e^{-i(a+b)}}{2i}$$

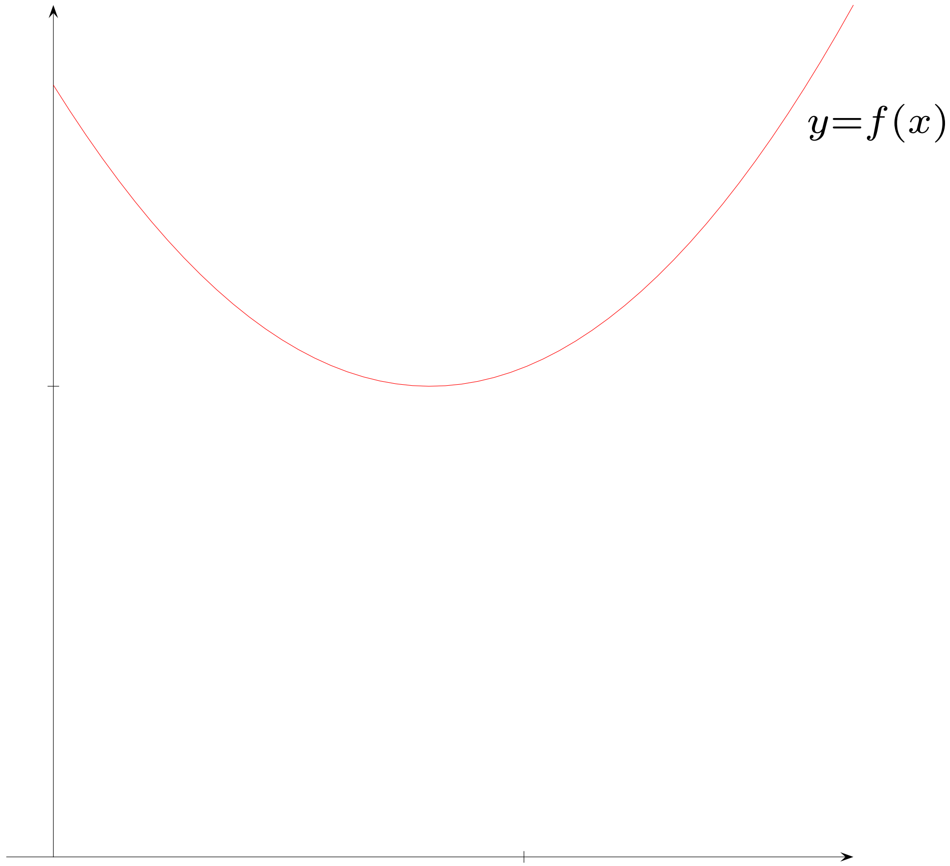
The addition formula

- $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$

- 

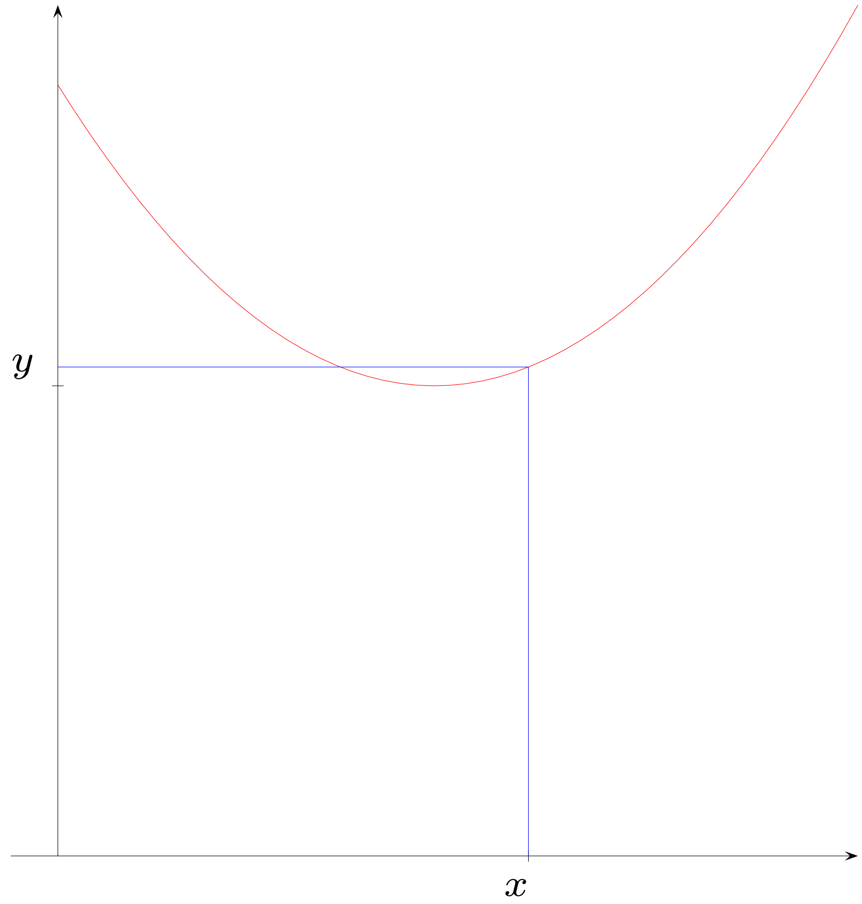
- $$\sin(a)\cos(b)+\cos(a)\sin(b) = \frac{e^{ia}-e^{-ia}}{2i}\frac{e^{ib}+e^{-ib}}{2} + \frac{e^{ia}+e^{-ia}}{2}\frac{e^{ib}-e^{-ib}}{2i}$$
$$= \frac{e^{i(a+b)}-e^{-i(a+b)}}{2i} = \sin(a+b)$$

Slopes



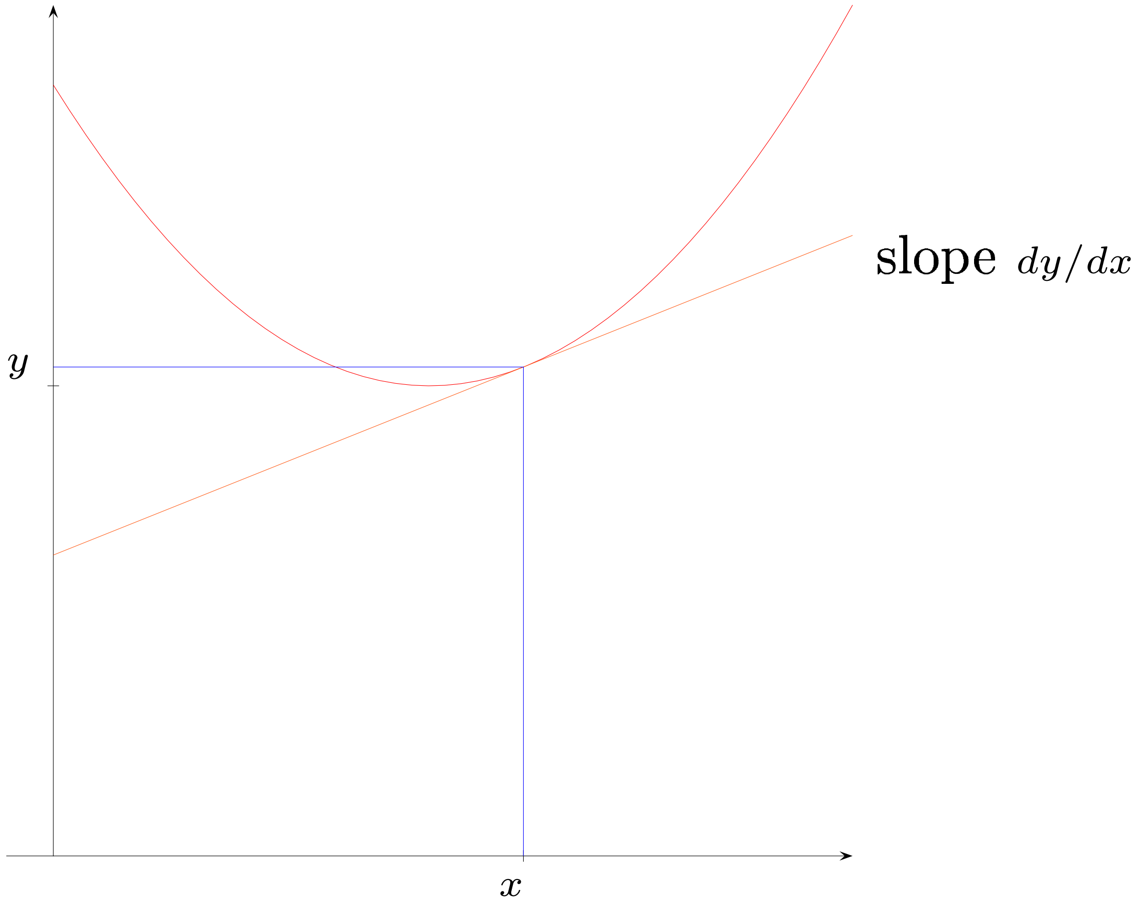
Consider variables x and y related by $y = f(x)$.

Slopes



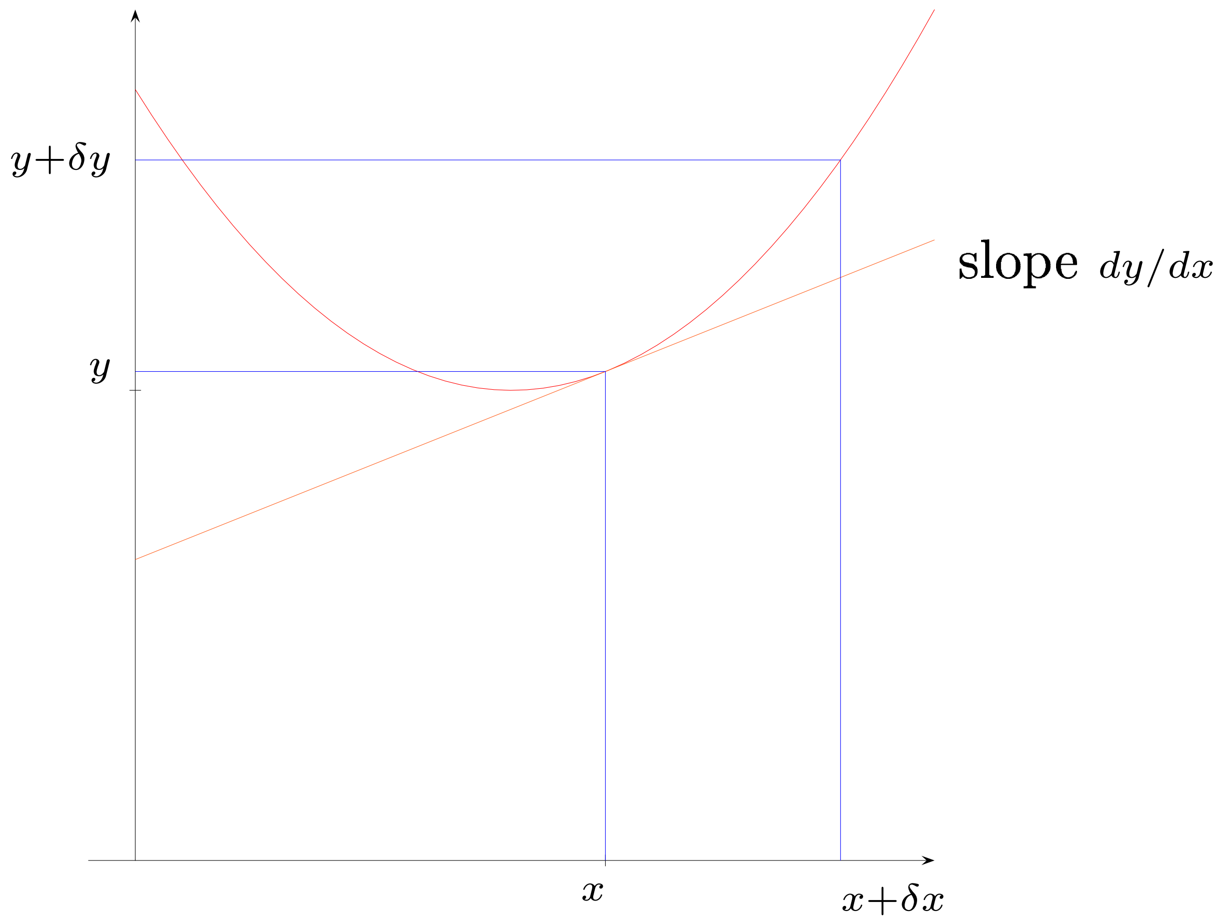
Consider variables x and y related by $y = f(x)$.

Slopes



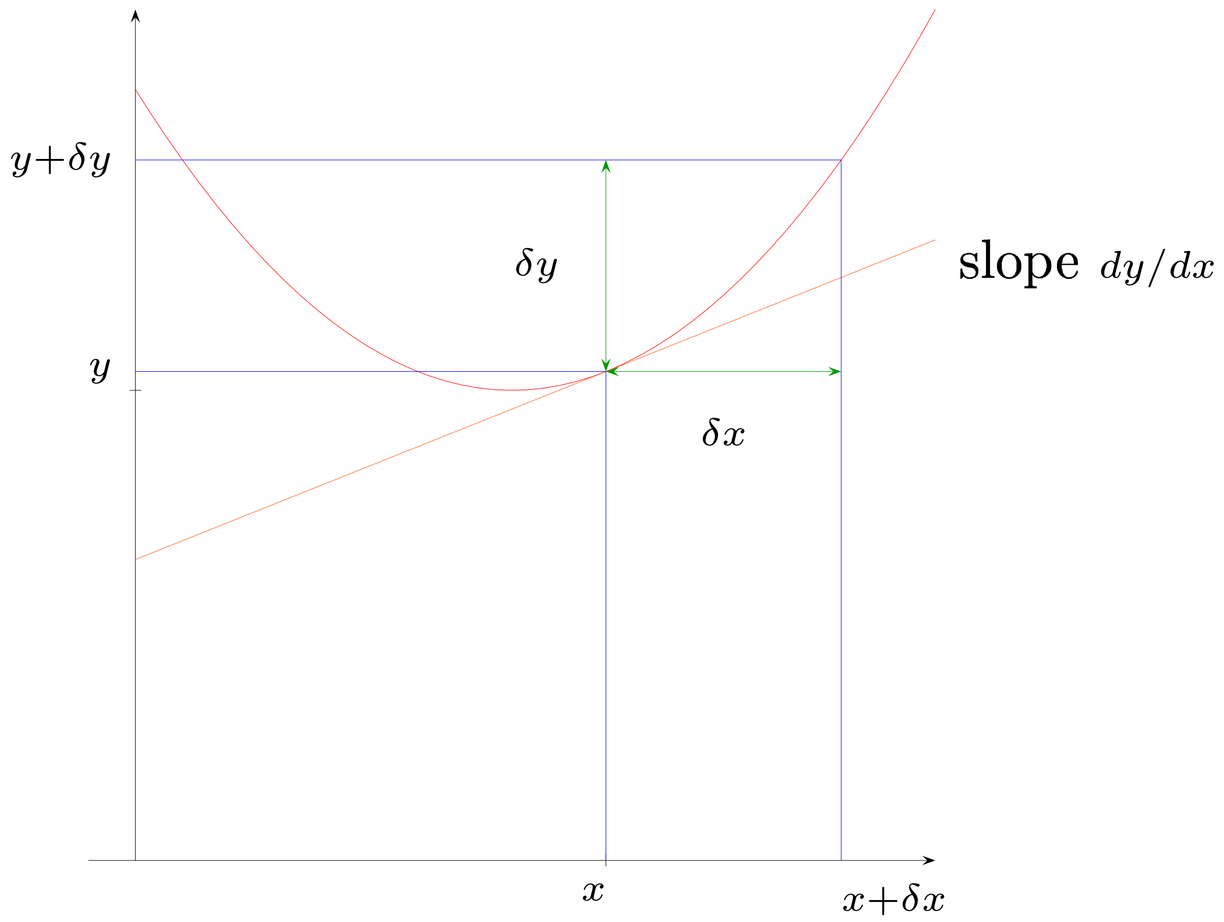
dy/dx is the slope of the tangent line to the graph.

Slopes



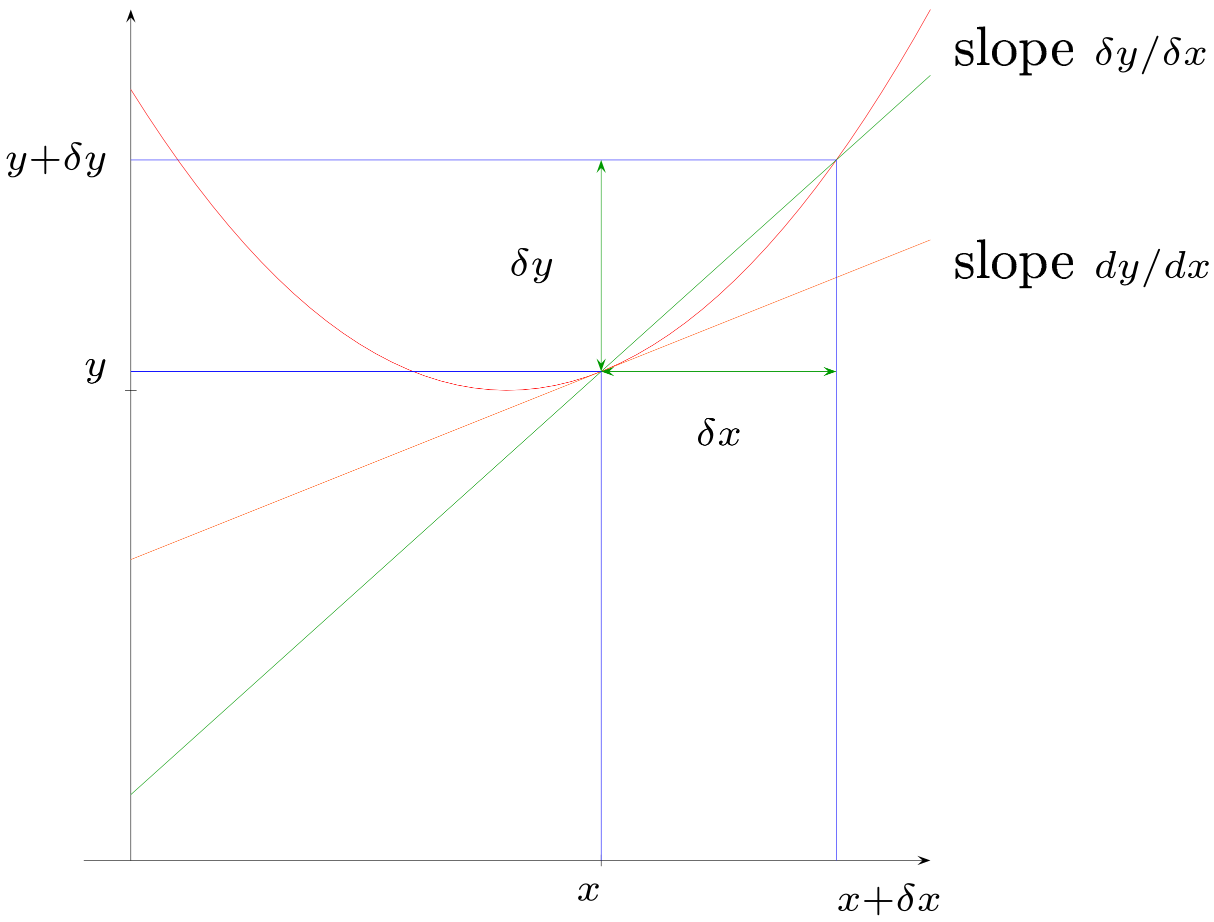
If x changes by a small amount δx, then y will change by a small amount δy.

Slopes



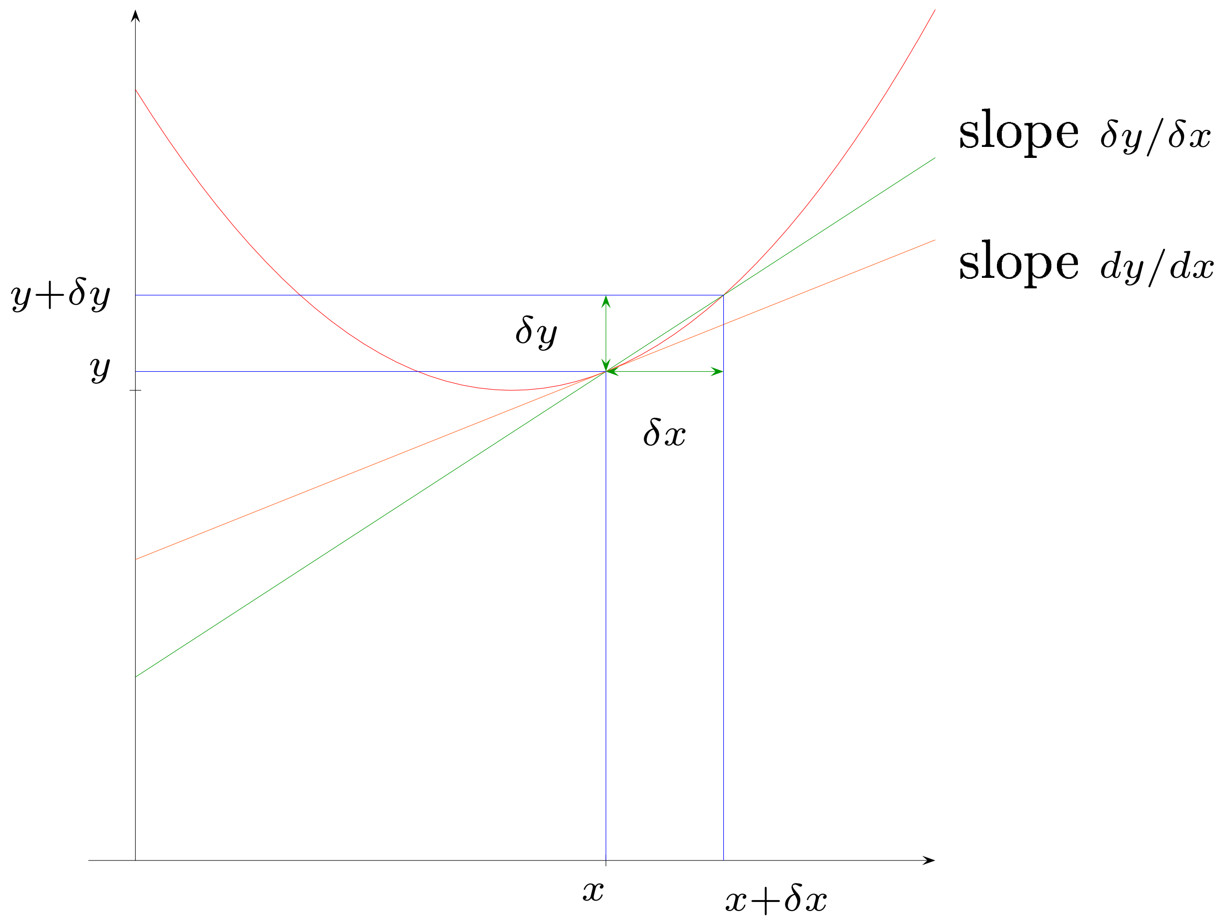
If x changes by a small amount δx, then y will change by a small amount δy.

Slopes



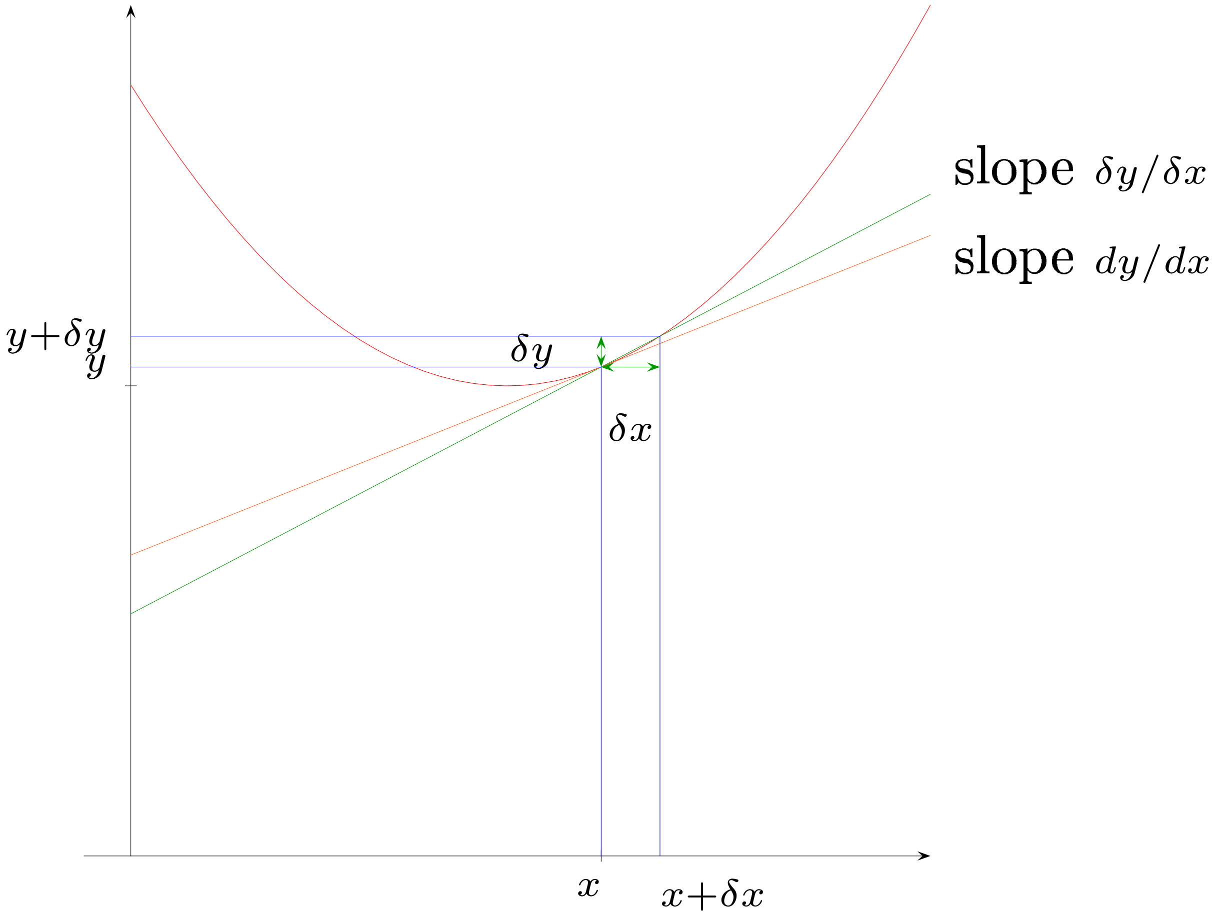
The ratio $\delta y/\delta x$ is the slope of a chord cutting across the graph.

Slopes



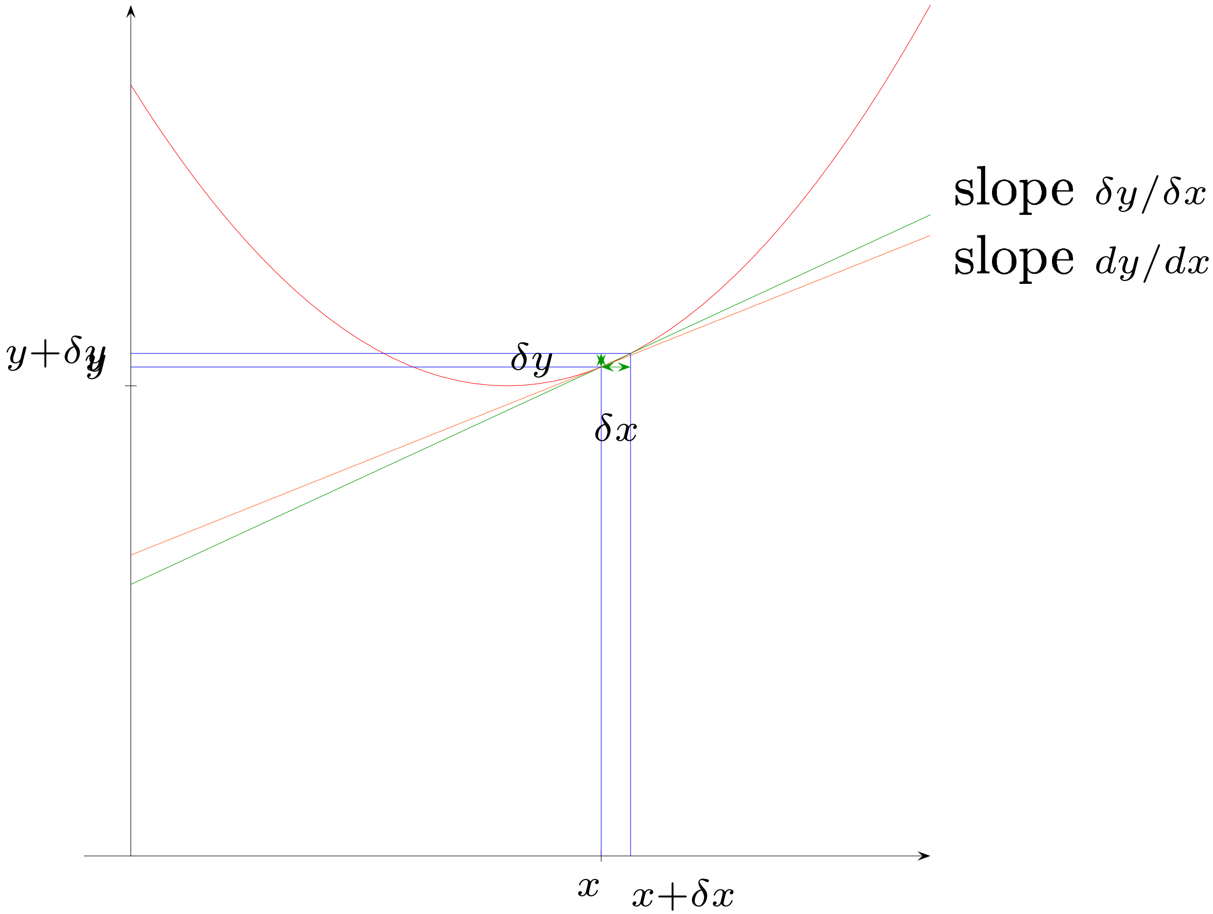
The slope of the chord changes slightly as δx decreases.

Slopes



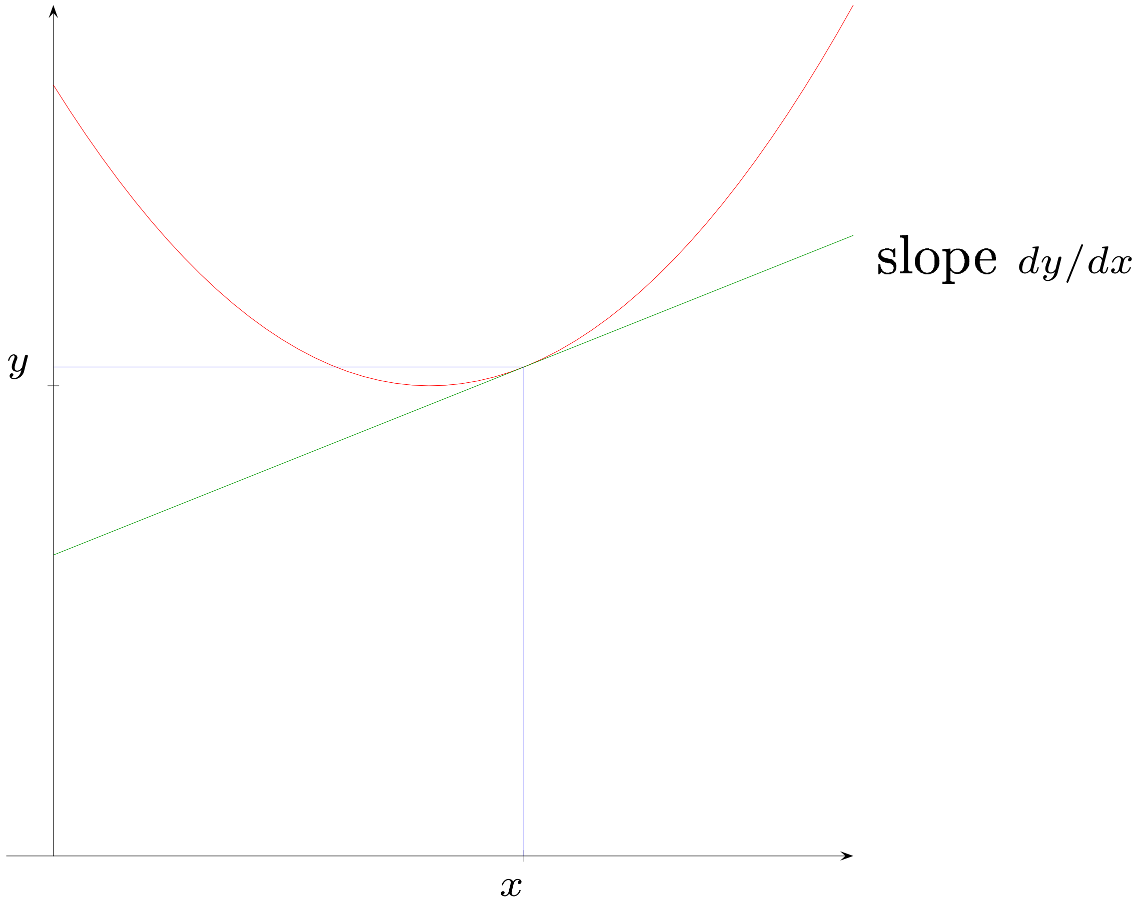
As δx approaches zero, the chord approaches the tangent, and $\delta y/\delta x$ approaches dy/dx.

Slopes



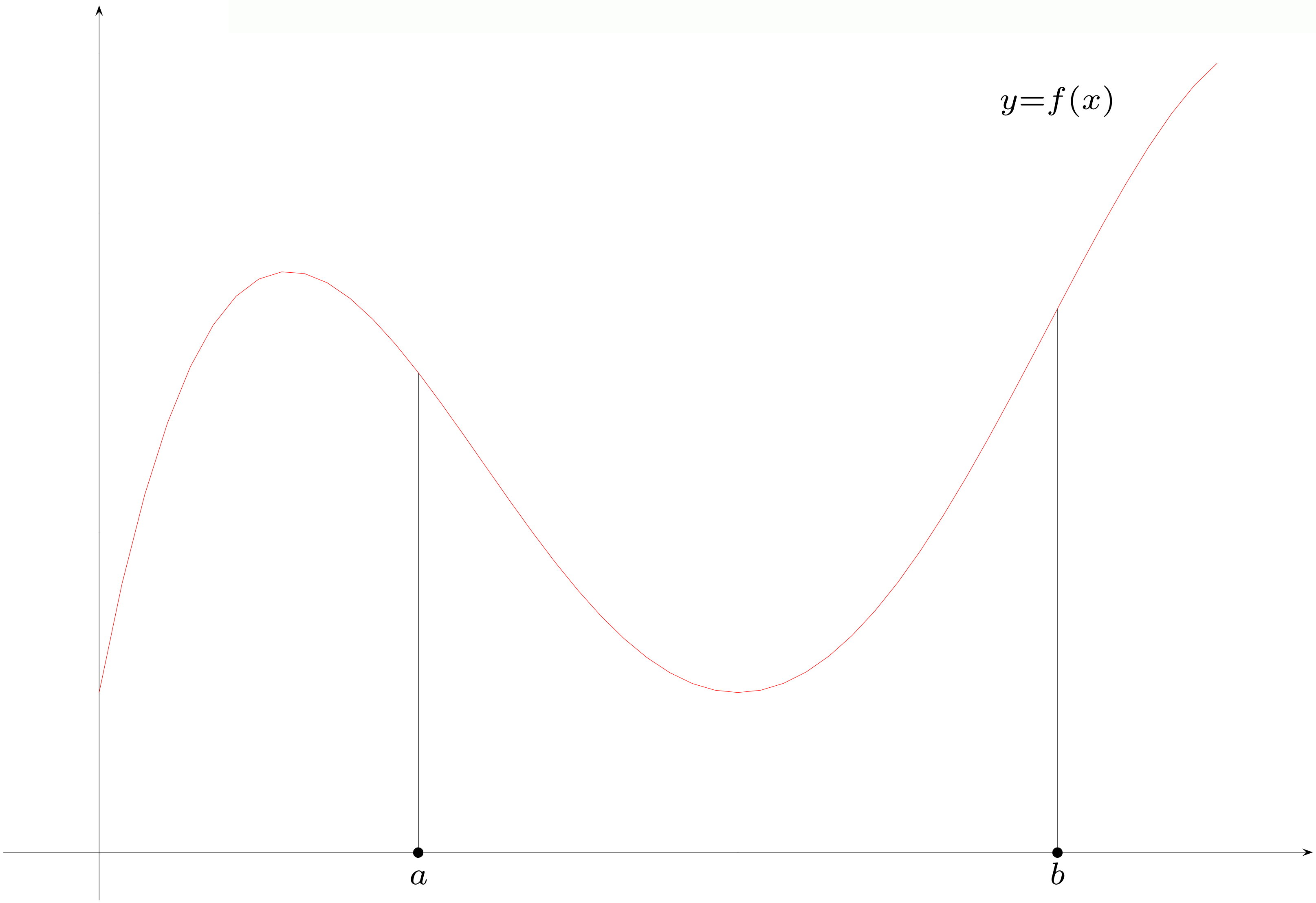
As δx approaches zero, the chord approaches the tangent, and $\delta y/\delta x$ approaches dy/dx.

Slopes



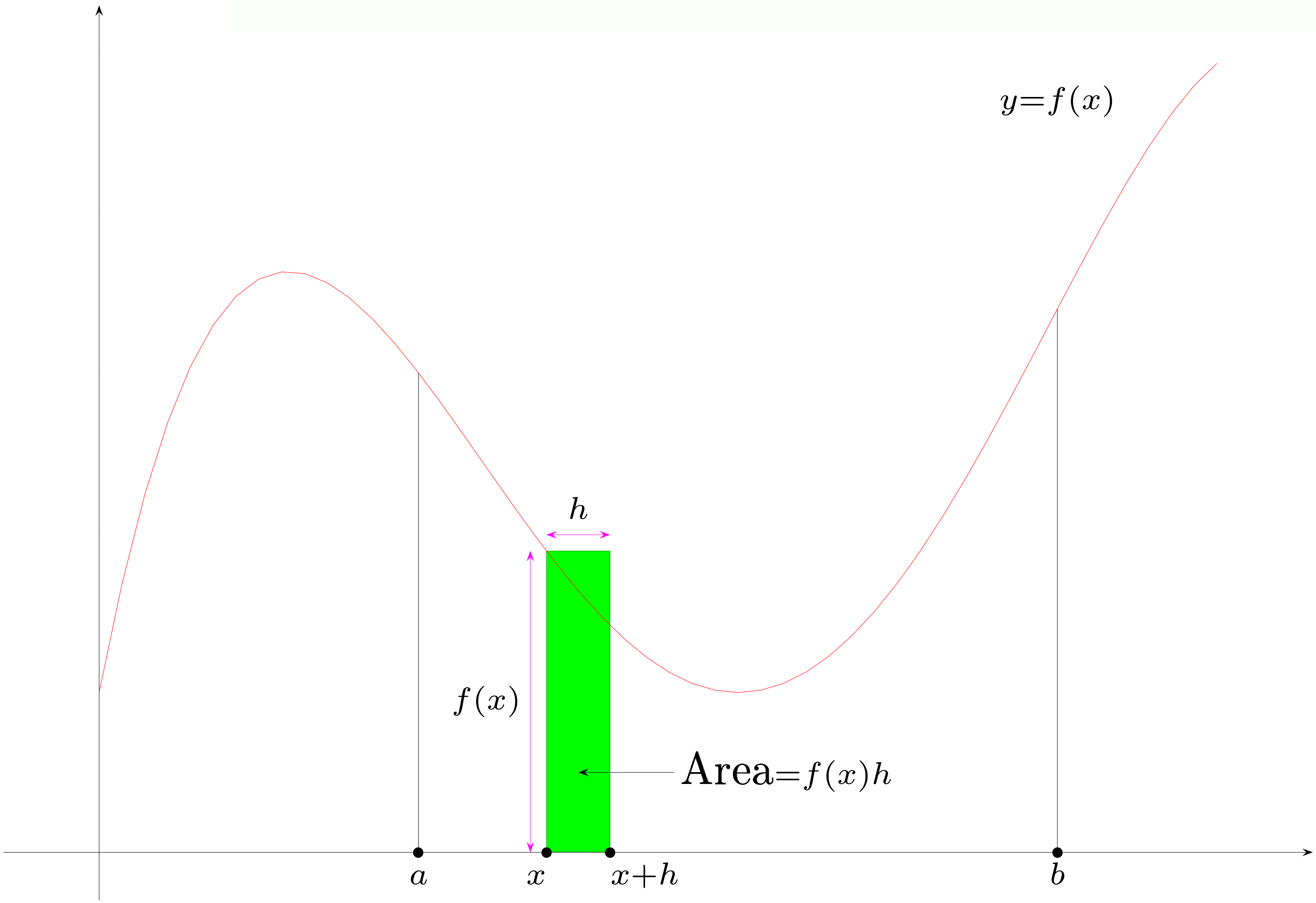
As δx approaches zero, the chord approaches the tangent, and $\delta y/\delta x$ approaches dy/dx.

Areas



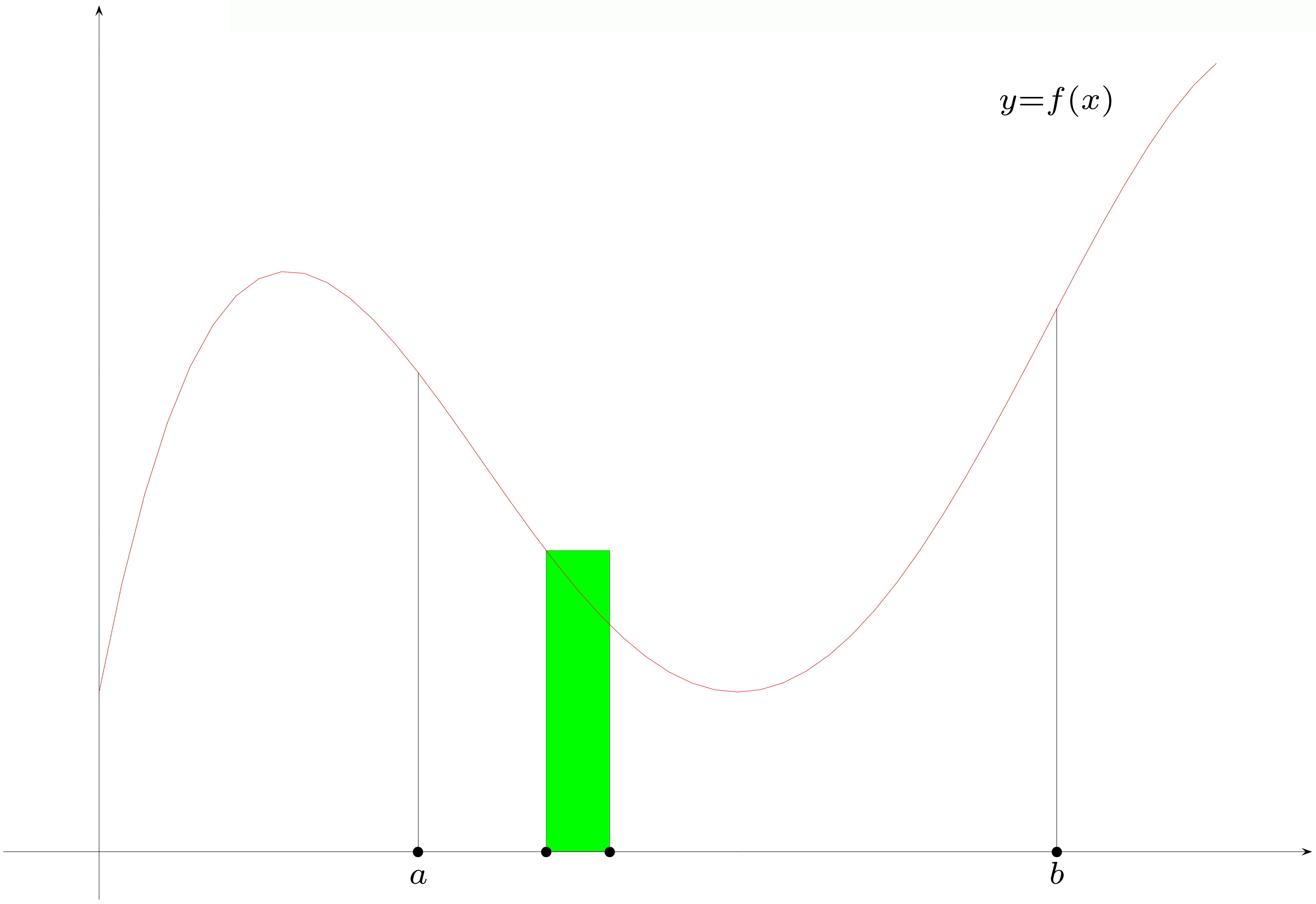
Consider the integral $\int_a^b f x)\, dx$.

Areas



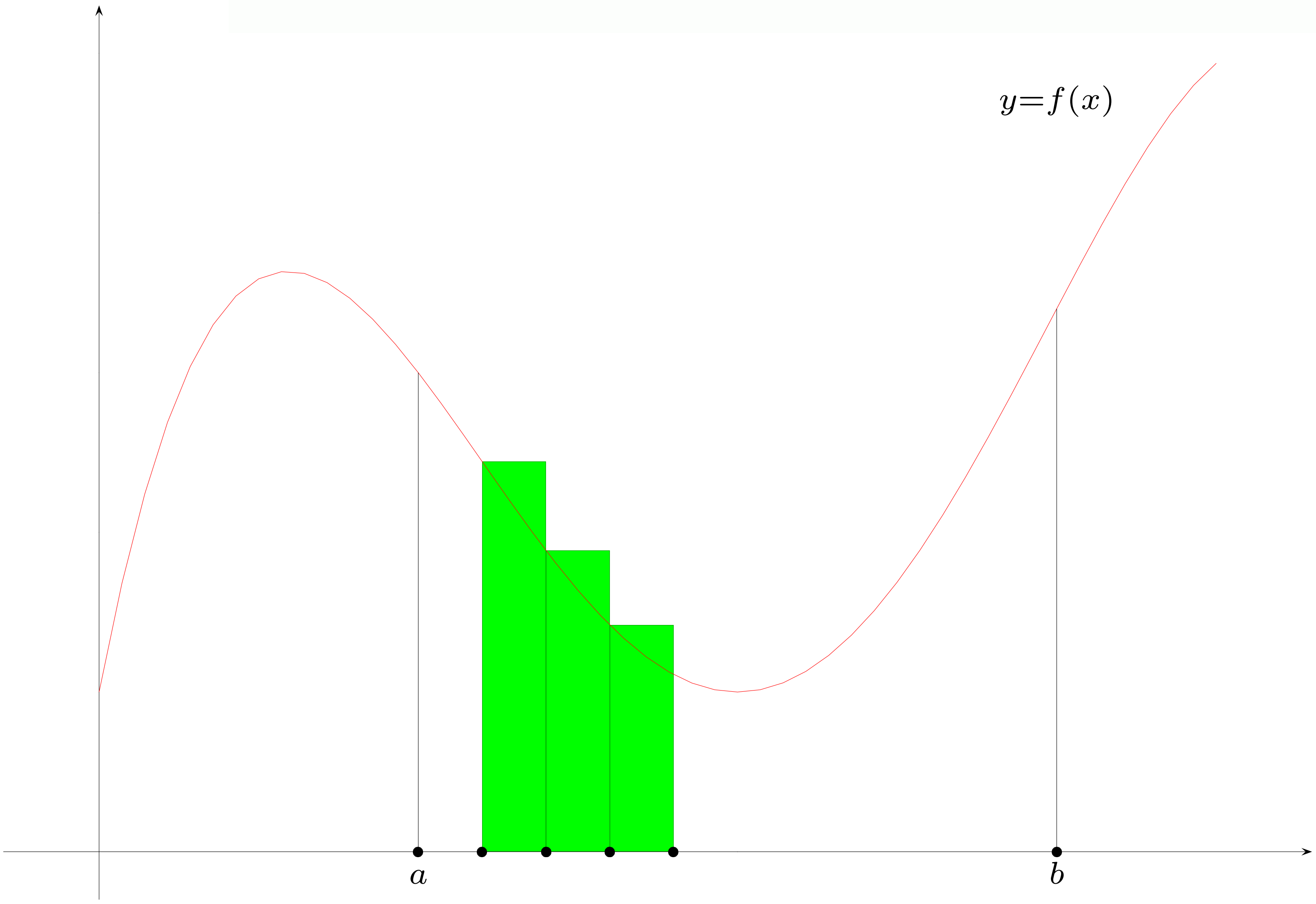
For each short interval $[x, x+h] \subset [a, b]$, we have a contribution $f(x)h$. This is the area of the green rectangle.

Areas



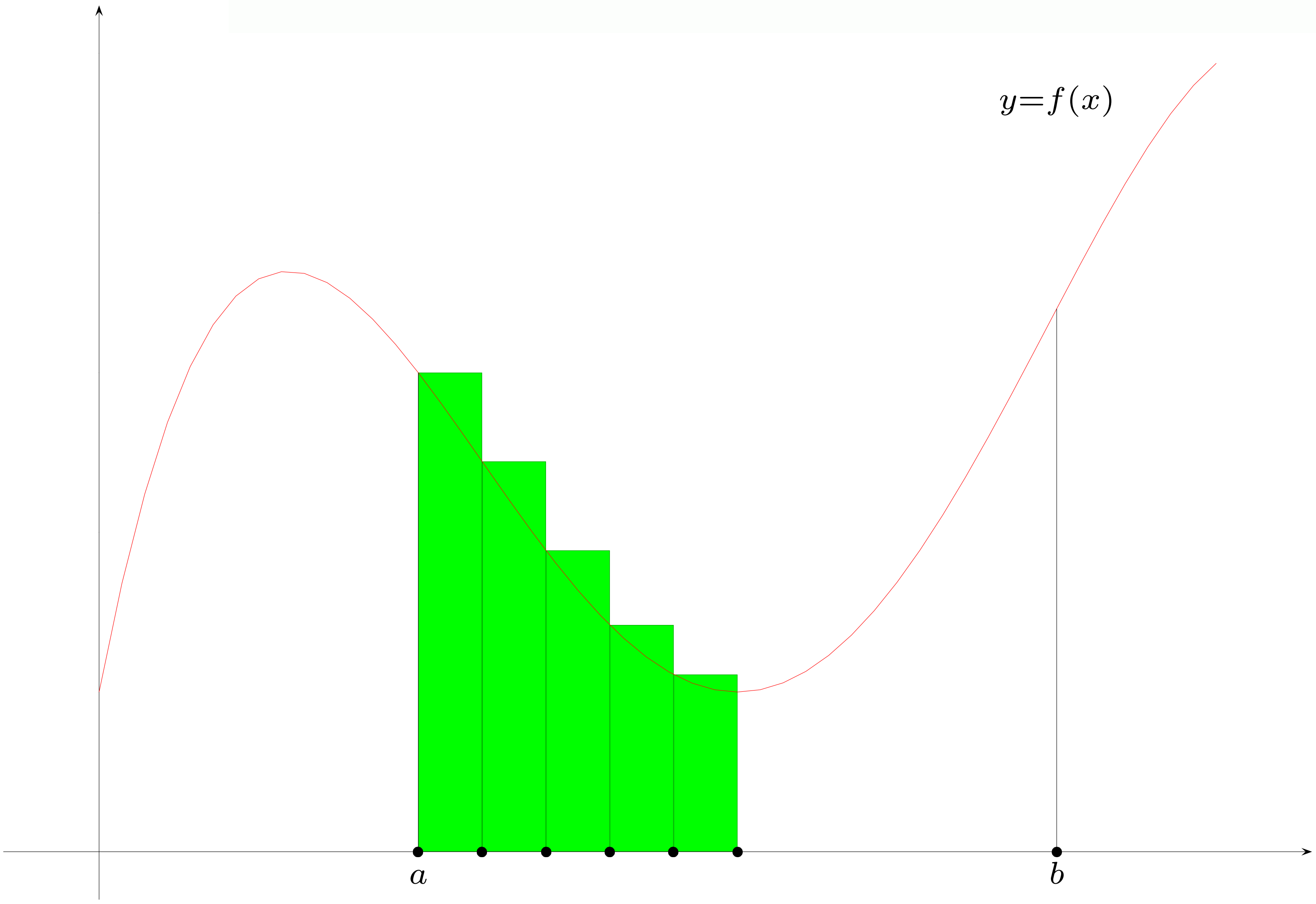
This is the contribution from one short interval, but we need to add together the contributions from many short intervals.

Areas



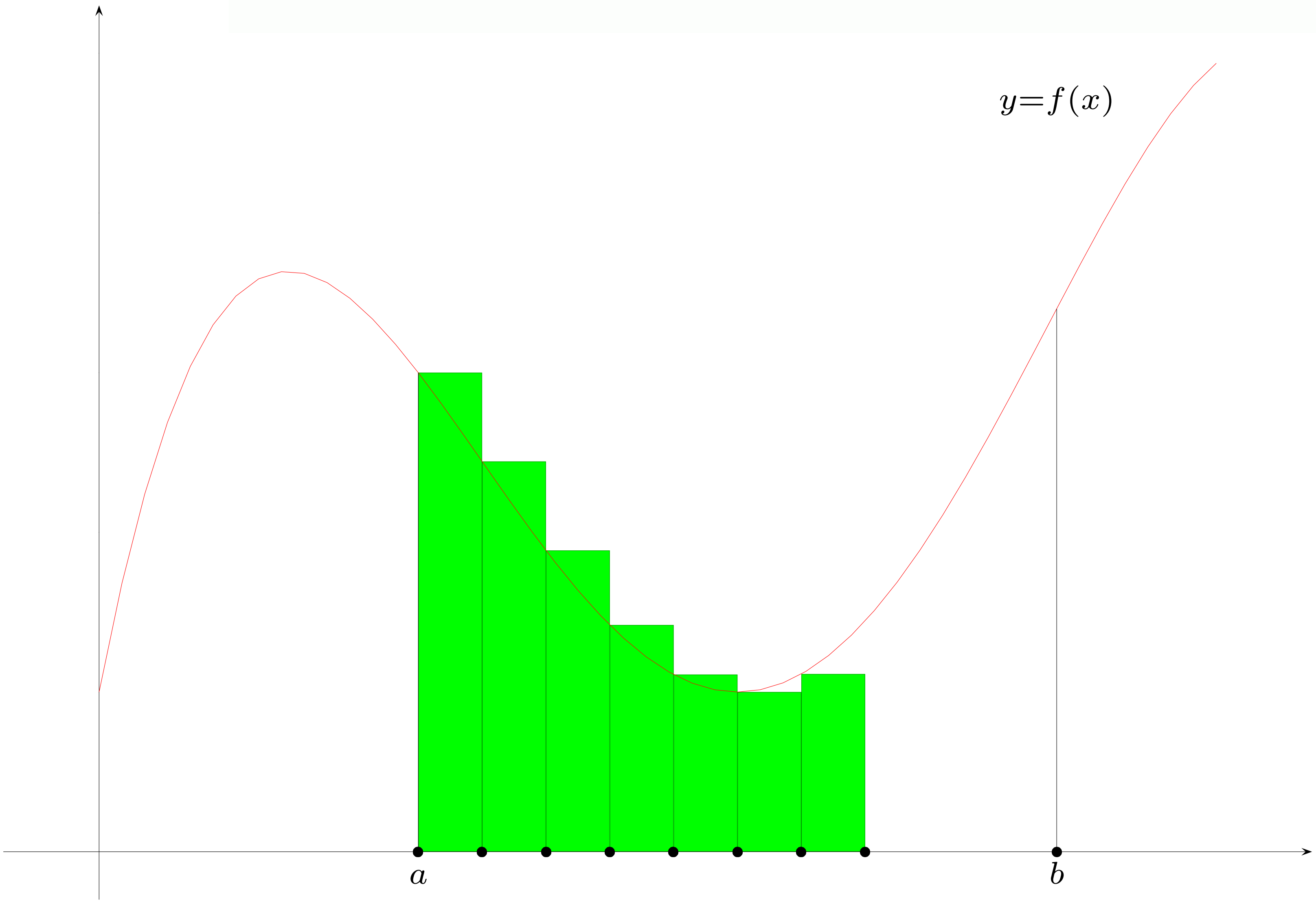
Here we have added in two more intervals

Areas



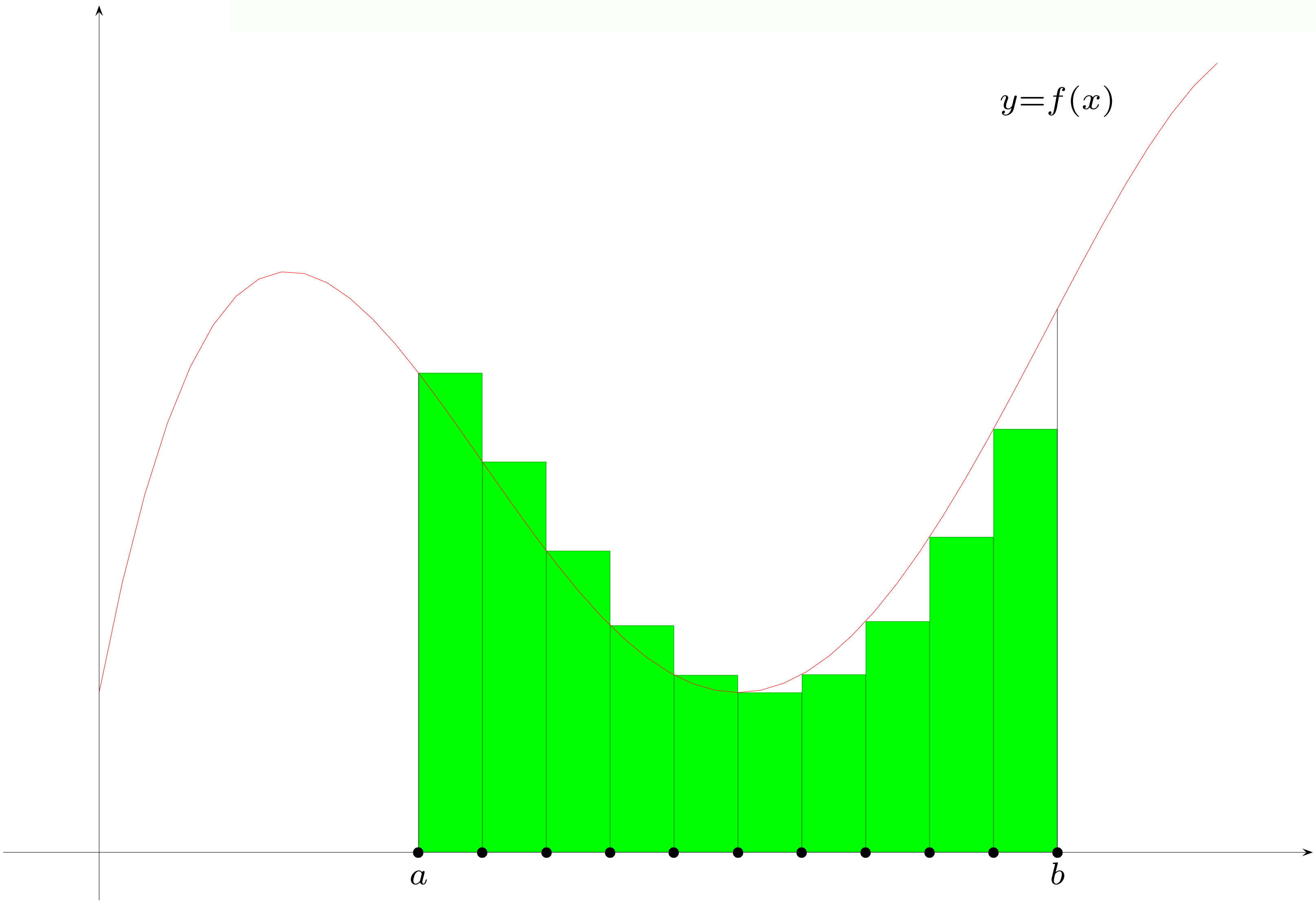
Here we have added in two more intervals — and two more

Areas



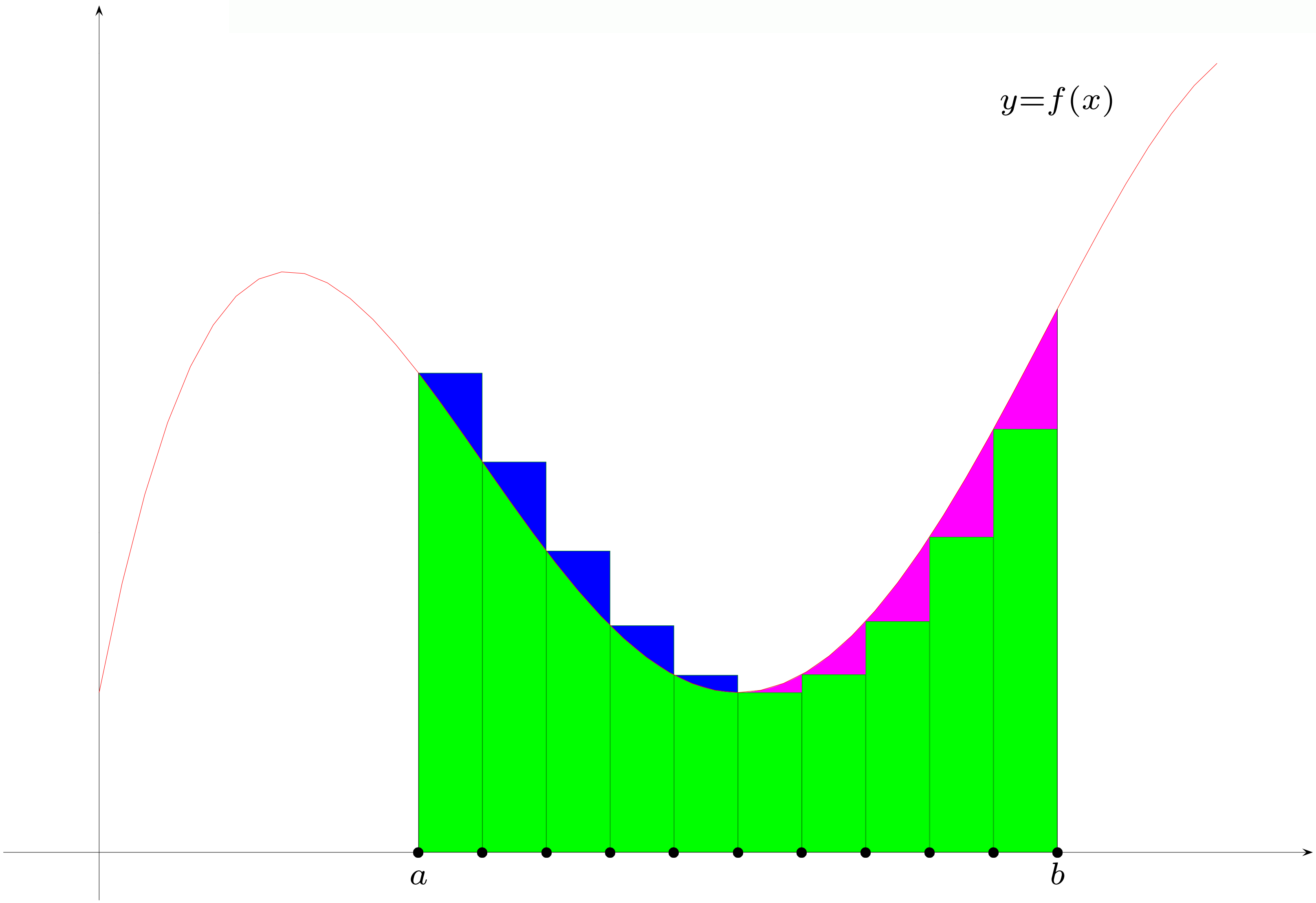
Here we have added in two more intervals — and two more — and two more

Areas



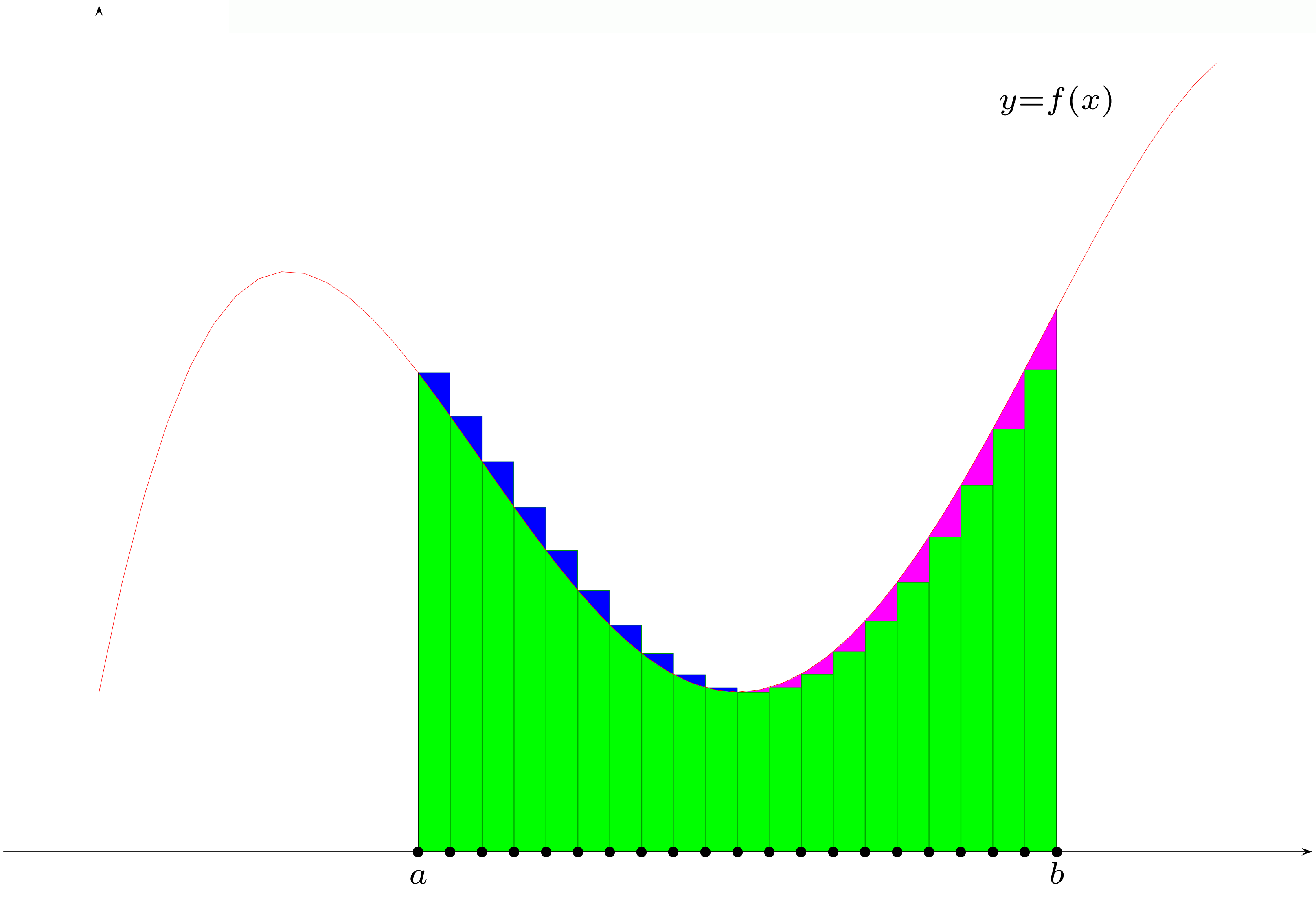
Now we have divided the whole interval $[a, b]$ into subintervals of length h. The sum of the terms $f(x)h$ is the area of the green region.

Areas



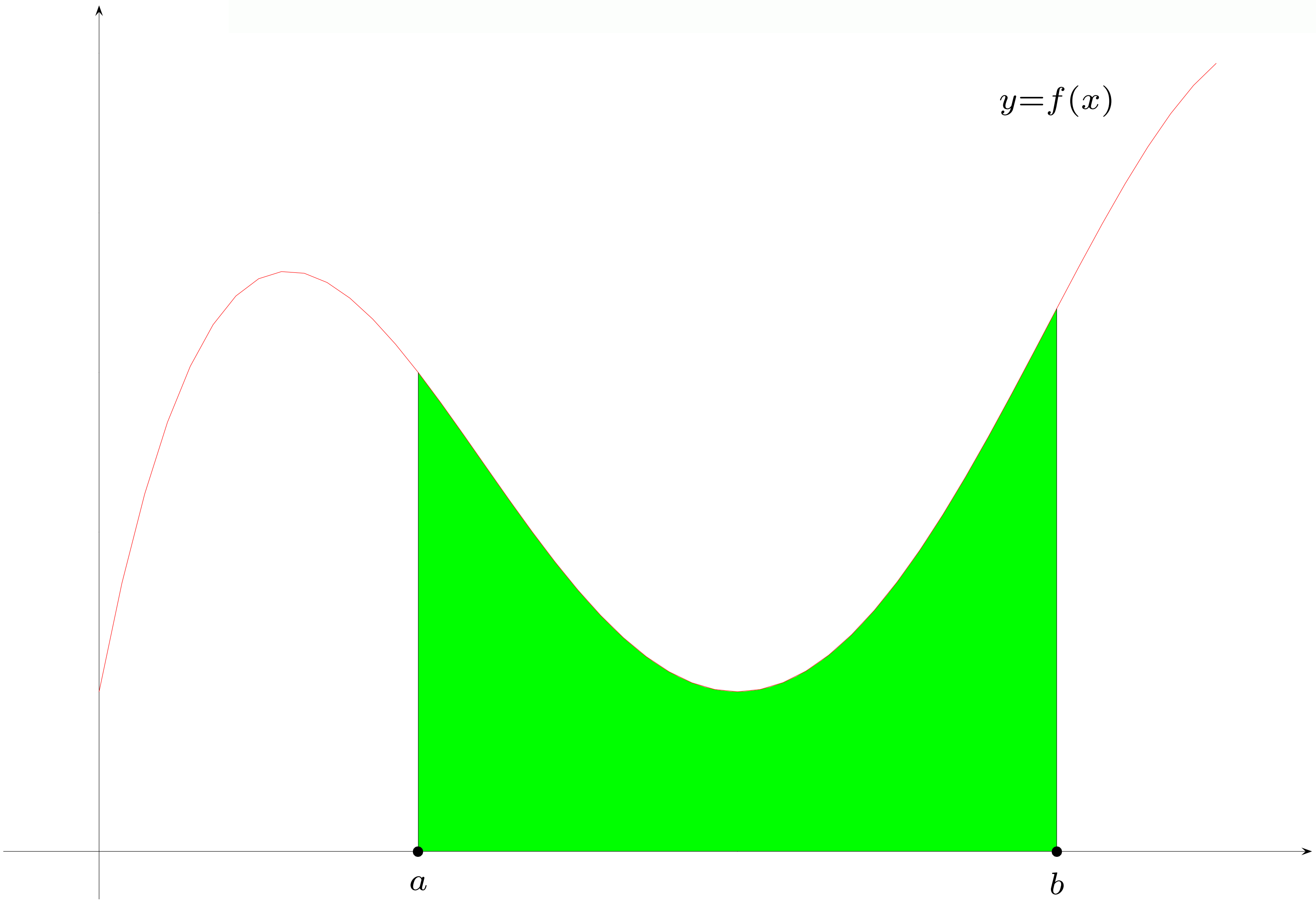
This is not exactly the same as the area under the curve, because of the regions marked in blue and pink.

Areas



However, the error decreases if we make h smaller

Areas



However, the error decreases if we make h smaller, and tends to zero in the limit.