

Proof assistants as a routine tool?

Neil Strickland

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- ▶ Homotopy type theory.
- ▶ Experimentation and semi-formal verification in new mathematics; but not formal verification.
- ▶ New interest creates new opportunities.

A plug for Freek Wiedijk

`http://www.cs.kun.nl/~freek/`

This is the personal home page of Freek Wiedijk.

In case you're looking for a way to reach me:

my snail-mail addresses are: Zandstraat 28-1, 1011 HL
Amsterdam (home) and: Postbus 9010, 6500 GL Nijmegen, or:
Room 01.17, Mercator 1, Toernooiveld 212, 6525 EC
Nijmegen (work)

my telephone numbers are 06-20422671 (mobile), 020-4289648 (home) and 024-
3652649 (work) and the fax number of my work is 024-3652728

my e-mail is freek@cs.ru.nl (if you want to make sure that your mail won't be eaten by
my spam filter, mention [free ultrafilters](#) in the subject line of your message)

For my American friends: the name Freek is pronounced like "Phrake". It's a perfectly ordinary Dutch
name (from Frederic), no reference to [freak](#) was ever intended. And "Wiedijk" is pronounced like
"Weedike".



(Catalogs, comparisons, history, overview.)

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- ▶ Isabelle etc: mentioned for completeness.

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- ▶ L^AT_EX:

```
\documentclass{amsart}
\newtheorem{theorem}{Theorem}

\begin{document}

\begin{theorem}
  For every natural number  $n$ , there is a prime  $p$  with  $p > n$ .
\end{theorem}
\begin{proof}
  Put  $D = \{d : d \mid n! + 1 \text{ and } d > 1\}$ . This
  contains  $n! + 1$  itself, so it is a nonempty set of natural numbers,
  so it has a smallest element, say  $p$ . If there were a number  $d$ 
  with  $1 < d < p$  such that  $d \mid p$ , then we would have
   $d \mid p \mid n! + 1$ , so  $d \in D$ , but also  $d < p$ , which is impossible
  as  $p$  was defined to be the smallest element in  $D$ . Thus, there
  cannot be any such number  $d$ , which means that  $p$  is prime. Next,
  note that the numbers  $1, 2, \dots, n$  all divide  $n!$  and so do not
  divide  $n! + 1$ , so  $p$  cannot be any of these numbers, so  $p > n$ .
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- ▶ A collection of undergraduate level proofs with detailed, line by line commentary.

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- ▶ Library source code.
- ▶ Other background: several large scale software systems in a wide variety of languages; extensive semi-formal verification in Maple and Mathematica.
- ▶ I proved in Agda and Coq that there are infinitely many primes. Both were extremely painful.

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- ▶ The whole proof is 826 lines long. There are no comments.
- ▶ As is typical with Coq proof scripts, one cannot easily see how the proof works without stepping through it in CoqIde.

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- ▶ Many questions about compatibility and conversion.

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- ▶ `Coq.ZArith.Znumtheory` in the standard library
- ▶ Building user contributions requires additional tools, fiddling with environment variables.

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- ▶ From a constructive proof that $\exists! n P(n)$, you can “obviously” extract the value of n . You have to spend a lot of time reading non-obvious parts of the reference manual to understand why this does not work, and how to reorganise to avoid the problem.

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- ▶ I have started writing an extractable proof in the style of `Coq.Arith.Wf_nat`, but have not finished.

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- ▶ The main thing that would have reduced the pain: comparable examples, heavily annotated.
- ▶ Final step: apply the above with $m = n! + 1$.
- ▶ One needs basic facts like $k!|n!$ when $0 \leq k \leq n$, and $k|n!$ when $0 < k \leq n!$. I spent 78 lines on these. It was not too painful, but it would be better if these facts were in `Coq.Arith.Factorial`.

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